STATE MACHINES IN TLA+
In the first lecture, I introduced state machines as a simple abstraction of digital systems.

You saw how a tiny C program can be viewed as a state machine.

In this lecture, you will see how that state machine can be described mathematically, and you will get your first glimpse of TLA⁺.
What language should we use to describe state machines?
State machines are a simple and powerful abstraction.

The way we described the next state for the simple program is neither precise nor is it practical for real systems.
State machines are a simple and powerful abstraction.

We need a precise, practical way to describe them.
State machines are a simple and powerful abstraction.

We need a precise, practical way to describe them.

This is neither precise nor practical:

```
if current value of pc equals "start"
    then next value of i in {0, 1, ..., 1000}
    next value of pc equals "middle"
else if current value of pc equals "middle"
    then next value of i equals current value of i + 1
    next value of pc equals "done"
else no next values
```

State machines are a simple and powerful abstraction.

We need a precise, practical way to describe them.

The way we described the next state for the simple program is neither precise nor is it practical for real systems.

[slide 6]
We need a language for describing state machines.

We need a precise language for describing state machines.

Asked what such a language should look like,
We need a language for describing state machines.

Most software engineers want one like their favorite programming language.

We need a precise language for describing state machines.

Asked what such a language should look like, most programmers and software engineers want one that’s a lot like their favorite programming language.
We need a language for describing state machines.

Most software engineers want one like their favorite programming language.

TLA+

We need a precise language for describing state machines.

Asked what such a language should look like, most programmers and software engineers want one that’s a lot like their favorite programming language.

TLA+ takes a different approach.
We need a language for describing state machines.

Most software engineers want one like their favorite programming language.

TLA+ uses ordinary, simple math.

We need a precise language for describing state machines.

Asked what such a language should look like, most programmers and software engineers want one that’s a lot like their favorite programming language.

TLA+ takes a different approach. It uses ordinary, simple math.
We need a language for describing state machines.

Most software engineers want one like their favorite programming language.

TLA+ uses ordinary, simple math.

Most software engineers find that a terrible and terrifying idea.

We need a precise language for describing state machines.

Asked what such a language should look like, most programmers and software engineers want one that’s a lot like their favorite programming language.

TLA+ takes a different approach. It uses ordinary, simple math.

This strikes most programmers and software engineers as a terrible idea—and probably a terrifying one.

[slide 11]
Here’s what the designers of this real-time operating system said in this paper:

Here’s what the designers of this real-time operating system said in this paper:

[slide 12]
Here’s what the designers of this real-time operating system said in this paper:

An industrial Case: Pitfalls and Benefits of Applying Formal Methods to the Development of a Network-Centric RTOS

Eric Verhulst, Gjalt de Jong, and Vitaliy Mezhuyev

*Formal Methods 2008, pages 411–418*
While we had an initial bias toward using language X,

I’m not going to tell you what that language was
While we had an initial bias toward using language X, in the end it was decided to use TLA$^+$. 

I’m not going to tell you what that language was in the end it was decided to use TLA$^+$. 

[slide 15]
While we had an initial bias toward using language X, in the end it was decided to use TLA+. Although the mathematical notation of the TLA+ language was first considered a hindrance versus the C-like language X,

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*not a hindrance, a major benefit*
While we had an initial bias toward using language X, in the end it was decided to use TLA+. Although the mathematical notation of the TLA+ language was first considered a hindrance versus the C-like language X, in the end it has proven to be a major benefit as it forced us to reason in a much more abstract way about the system.

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in the end it has proven to be a major benefit

not a hindrance, a major benefit

as it forced us to reason in a much more abstract way about the system.

A more abstract way. And remember...
Remember what Brannon Batson said:

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The hard part of learning to write TLA+ specs is learning to think abstractly about the system.

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The hard part of learning to write TLA+ specs is learning to *think abstractly* about the system.
Remember what Brannon Batson said:

The hard part of learning to write TLA\(^+\) specs is learning to think abstractly about the system.

Being able to think abstractly improves our design process.

[slide 22]
Remember what Verhulst said:

And remember what Eric Verhulst, the leader of that real-time operating system project, said:
Remember what Verhulst said:

We witnessed first hand the brain washing done by years of C programming.

And remember what Eric Verhulst, the leader of that real-time operating system project, said:

We witnessed first hand the brain washing done by years of C programming.
Describing a state machine with math.
Our example C program:

```c
int i;
void main()
{
    i = someNumber();
    i = i + 1;
}
```

Remember our example C program.
Our example C program:

```c
int i;
void main()
{
    i = someNumber();
    i = i + 1;
}
```

We introduced \( pc \) to describe the control state.

Remember our example C program.

Recall that we introduced the variable \( pc \) to describe the control state.
Our example C program:

```c
int i;
void main()

pc = "start" { i = someNumber();
   i = i + 1;
}
```

We introduced $pc$ to describe the control state.

Remember our example C program.

Recall that we introduced the variable $pc$ to describe the control state.

$pc$ equals the string start means this is the next statement to be executed.
Our example C program:

```c
int i;
void main()
{
    i = someNumber();
    pc = "middle"
    i = i + 1;
}
```

We introduced $pc$ to describe the control state.

Remember our example C program.

Recall that we introduced the variable $pc$ to describe the control state.

$pc$ equals the string `start` means this is the next statement to be executed.

$pc$ equals `middle` means control is here.
Our example C program:

```c
int i;
void main()
{
    i = someNumber();
    i = i + 1;
}  pc = “done”
```

We introduced \( pc \) to describe the control state.

Remember our example C program.
Recall that we introduced the variable \( pc \) to describe the control state.

\( pc \) equals the string \( start \) means this is the next statement to be executed.

\( pc \) equals \( middle \) means control is here.

and \( pc \) equals \( done \) when execution has terminated.

[slide 30]
int i;
void main()
{
    i = someNumber();
    i = i + 1;
}

To describe this program,
int i;
void main()
{
    i = someNumber();
    i = i + 1;
}

We must describe:

To describe this program, we must describe two things:
int i;
void main()
{
    i = someNumber();
    i = i + 1;
}

We must describe:

1. Possible initial values of variables.

To describe this program, we must describe two things:

The possible initial values of the variables.
We must describe:

1. Possible initial values of variables.
2. The relation between their values in the current state and their possible values in the next state.

To describe this program, we must describe two things:

The possible initial values of the variables.

And what the relation is between the values of the variables in the current state and their possible values in the next state.
int i;
void main()
{
    i = someNumber();
    i = i + 1;
}

We must describe:

1. **Possible initial values of variables.**

2. The relation between their values in the current state and their possible values in the next state.

To describe this program, we must describe two things:

The possible initial values of the variables.

And what the relation is between the values of the variables in the current state and their possible values in the next state.

Let’s start with the initial values.
Possible initial values of variables.

Must replace "and" by a mathematical operator. Written && in some programming languages. Written $\land$ in mathematics. Some unnecessary parentheses make it easier to read. These are the initial values. But we want a mathematical formula, so we must replace and by a mathematical operator.
Possible initial values of variables.

\[ i = 0 \text{ and } pc = "start" \]

These are the initial values. But we want a mathematical formula, so
Possible initial values of variables.

\[ i = 0 \text{ and } pc = \text{“start”} \]

Must replace “and” by a mathematical operator.

These are the initial values. But we want a mathematical formula, so we must replace \textit{and} by a mathematical operator.
Possible initial values of variables.

\[ i = 0 \text{ and } pc = "start" \]

Must replace “and” by a mathematical operator.

Written `&&` in some programming languages.

That operator is written `ampersand ampersand` in some programming languages.
Possible initial values of variables.

\[ i = 0 \land pc = "\text{start}" \]

Must replace “and” by a mathematical operator.

Written \&\& in some programming languages.  
Written \land in mathematics.

That operator is written \textit{ampersand ampersand} in some programming languages.

It's written with this symbol in mathematics.
Possible initial values of variables.

\((i = 0) \land (pc = "start")\)

Must replace “and” by a mathematical operator.

Written `&&` in some programming languages.

Written `\land` in mathematics.

Some unnecessary parentheses make it easier to read.

That operator is written `ampersand ampersand` in some programming languages.

It’s written with this symbol in mathematics.

Let’s add some unnecessary parentheses to make it easier to read.
2. The relation between their values in the current state and their possible values in the next state.

```c
int i;
void main()
{
    i = someNumber();
    i = i + 1;
}
```

Now, let's describe the relation between the values of the variables in the current state and their possible values in the next state.
2. The relation between their values in the current state and their possible values in the next state.

```c
int i;
void main()
{
    i = someNumber();
    i = i + 1;
}
```

- **if** current value of `pc` equals "start"
  - **then** next value of `i` in \{0, 1, \ldots, 1000\}
  - next value of `pc` equals "middle"
- **else if** current value of `pc` equals "middle"
  - **then** next value of `i` equals current value of `i + 1`
  - next value of `pc` equals "done"
- **else** no next values

Now, let’s describe the relation between the values of the variables in the current state and their possible values in the next state.

Here’s how I did it in the previous lecture.
if current value of \( pc \) equals "start"
    then next value of \( i \) in \( \{0, 1, \ldots, 1000\} \)
        next value of \( pc \) equals "middle"
else if current value of \( pc \) equals "middle"
    then next value of \( i \) equals current value of \( i + 1 \)
        next value of \( pc \) equals "done"
else no next values

OK. Let’s now write this in math.
if current value of $pc$ equals "start"
  then next value of $i$ in \{0, 1, \ldots, 1000\}
  next value of $pc$ equals "middle"
else if current value of $pc$ equals "middle"
  then next value of $i$ equals current value of $i + 1$
  next value of $pc$ equals "done"
else no next values

Let's write this in mathematics.

OK. Let's now write this in math.
if current value of \( pc \) equals "start"
    then next value of \( i \) in \{0, 1, \ldots, 1000\}
    next value of \( pc \) equals "middle"
else if current value of \( pc \) equals "middle"
    then next value of \( i \) equals current value of \( i + 1 \)
    next value of \( pc \) equals "done"
else no next values

Let’s write this in mathematics.
This requires some notation.

OK. Let’s now write this in math.
This requires replacing words with some notation.
if current value of $pc$ equals "start"
  then next value of $i$ in $\{0, 1, \ldots, 1000\}$
  next value of $pc$ equals "middle"
else if current value of $pc$ equals "middle"
  then next value of $i$ equals current value of $i + 1$
  next value of $pc$ equals "done"
else no next values

OK. Let’s now write this in math.

This requires replacing words with some notation.

First, let’s get rid of “current value of”
if current value of $pc$ equals "start"
then next value of $i$ in \{0, 1, \ldots, 1000\}
next value of $pc$ equals "middle"
else if current value of $pc$ equals "middle"
then next value of $i$ equals current value of $i + 1$
next value of $pc$ equals "done"
else no next values

$pc$ means current value of $pc$

OK. Let’s now write this in math.
This requires replacing words with some notation.
First, let’s get rid of “current value of”

We simply let $pc$ mean the current value of $pc$
and let $i$ mean the current value of $i$
if current value of $pc$ equals "start"
then next value of $i$ in $\{0, 1, \ldots, 1000\}$
next value of $pc$ equals "middle"
else if current value of $pc$ equals "middle"
then next value of $i$ equals current value of $i + 1$
next value of $pc$ equals "done"
else no next values

$p c$ means current value of $pc$
i means current value of $i$

OK. Let’s now write this in math.

This requires replacing words with some notation.

First, let’s get rid of “current value of”

We simply let $pc$ mean the current value of $pc$
and let $i$ mean the current value of $i$
if \( pc \) equals \("start"\)
then next value of \( i \) in \( \{0, 1, \ldots, 1000\} \)
next value of \( pc \) equals \("middle"\)
else if \( pc \) equals \("middle"\)
then next value of \( i \) equals \( i + 1 \)
next value of \( pc \) equals \("done"\)
else no next values

Next, we get rid of \"next value of\"
if $pc$ equals "start"
    then next value of $i$ in $\{0, 1, \ldots, 1000\}$
    next value of $pc$ equals "middle"
else if $pc$ equals "middle"
    then next value of $i$ equals $i + 1$
    next value of $pc$ equals "done"
else no next values

Next, we get rid of "next value of"
if $pc$ equals "start"
  then next value of $i$ in \{0, 1, \ldots, 1000\}
  next value of $pc$ equals "middle"
else if $pc$ equals "middle"
  then next value of $i$ equals $i + 1$
  next value of $pc$ equals "done"
else no next values

$p_{c}'$ means next value of $pc$
$i_{i}'$ means next value of $i$

Next, we get rid of “next value of”

by letting $pc$ prime and $i$ prime mean the next values of $pc$ and $i$
if $pc$ equals "start"
  then $i'$ in $\{0, 1, \ldots, 1000\}$
    $pc'$ equals "middle"
  else if $pc$ equals "middle"
    then $i'$ equals $i + 1$
      $pc'$ equals "done"
  else no next values

Next, we get rid of "next value of"
by letting $pc$ prime and $i$ prime mean the next values of $pc$ and $i$

And finally, we replace the word "equals" by an equal sign.
if \( pc \) equals “start”

then \( i' \) in \( \{0, 1, \ldots, 1000\} \)

\( pc' \) equals “middle”

else if \( pc \) equals “middle”

then \( i' \) equals \( i + 1 \)

\( pc' \) equals “done”

else no next values

equals \( \rightarrow \) =

Next, we get rid of “next value of”

by letting \( pc \) prime and \( i \) prime mean the next values of \( pc \) and \( i \)

And finally, we replace the word “equals” by an equal sign.
if \( pc = \text{"start"} \)

  then \( i' \) in \( \{0, 1, \ldots, 1000\} \)

  \( pc' = \text{"middle"} \)

else if \( pc = \text{"middle"} \)

  then \( i' = i + 1 \)

  \( pc' = \text{"done"} \)

else no next values

Next, we get rid of “next value of”

by letting \( pc \) prime and \( i \) prime mean the next values of \( pc \) and \( i \)

And finally, we replace the word “equals” by an equal sign.

Whew!

[slide 55]
if $pc = \text{"start"}$
   then $i' \in \{0, 1, \ldots, 1000\}$
       $pc' = \text{"middle"}$
else if $pc = \text{"middle"}$
   then $i' = i + 1$
       $pc' = \text{"done"}$
else no next values

It’s now easier to read.

It’s now a lot easier to read.
if $pc = \text{“start”}$
    then $i' \in \{0, 1, \ldots, 1000\}$
        $pc' = \text{“middle”}$
else if $pc = \text{“middle”}$
    then $i' = i + 1$
        $pc' = \text{“done”}$
else no next values

It’s now easier to read.

But it’s not yet mathematics.

It’s now a lot easier to read.

But it’s not yet a mathematical formula.
if $pc = \text{"start"}$
  then $i'$ in $\{0, 1, \ldots, 1000\}$
    $pc' = \text{"middle"}$
  else if $pc = \text{"middle"}$
    then $i' = i + 1$
      $pc' = \text{"done"}$
  else no next values

in here means is an element of the set of integers from 0 to 1000.
if \( pc = "start" \) is an element of the set
then \( i' \) in \( \{0, 1, \ldots, 1000\} \)
\( pc' = "middle" \)
else if \( pc = "middle" \)
then \( i' = i + 1 \)
\( pc' = "done" \)
else no next values

\textbf{in} here means \( \) is an element of the set of integers from 0 to 1000.
if $pc = \text{"start"}$ then $i' \in \{0, 1, \ldots, 1000\}$

$p'c$ = \text{“middle"}

else if $pc = \text{“middle"}$

then $i' = i + 1$

$p'c$ = \text{“done"}

else no next values

Written in math as $\in$.

in here means is an element of the set of integers from 0 to 1000.

$\in$ is written in mathematics as this symbol.
if \( pc = \text{“start”} \)

then \( i' \in \{0, 1, \ldots, 1000\} \)

\( pc' = \text{“middle”} \)

else if \( pc = \text{“middle”} \)

then \( i' = i + 1 \)

\( pc' = \text{“done”} \)

else no next values

\textit{In here means is an element of the set of integers from 0 to 1000.} 

\textit{In} is written in mathematics as this symbol.
if $pc = "start"$
   then $i' \in \{0, 1, \ldots, 1000\}$
       $pc' = "middle"$
   else if $pc = "middle"$
       then $i' = i + 1$
           $pc' = "done"$
   else no next values

"\ldots" is informal math.

in here means is an element of the set of integers from 0 to 1000.

\textit{In} is written in mathematics as this symbol.

Dot-dot-dot is informal math. We want to write this whole formula in a precisely defined language.
if \( pc = "start" \)

\[
\text{then } i' \in \{0, 1, \ldots, 1000\} \\
\text{pc'} = "middle"
\]

else if \( pc = "middle" \)

\[
\text{then } i' = i + 1 \\
\text{pc'} = "done"
\]

else  no next values

This set

\[ i \in \{0, 1, \ldots, 1000\} \]

in here means is an element of the set of integers from 0 to 1000.

In is written in mathematics as this symbol.

Dot-dot-dot is informal math. We want to write this whole formula in a precisely defined language.

The set of integers from 0 to 1000 is written in TLA+ like this.
if \( pc = \text{"start"} \) 
  then \( i' \in 0..1000 \) 
      \( pc' = \text{"middle"} \) 
  else if \( pc = \text{"middle"} \) 
    then \( i' = i + 1 \) 
        \( pc' = \text{"done"} \) 
  else no next values

This set is written in TLA\(^+\) as \( 0..1000 \).

in here means is an element of the set of integers from 0 to 1000.

\( \in \) is written in mathematics as this symbol.

Dot-dot-dot is informal math. We want to write this whole formula in a precisely defined language.

The set of integers from 0 to 1000 is written in TLA+ like this.
if $pc = \text{"start"}$

then $i' \in 0..1000$

$pc' = \text{"middle"}$

else if $pc = \text{"middle"}$

then $i' = i + 1$

$pc' = \text{"done"}$

else no next values

This set is written in TLA+ as $0..1000$.

The operator $\ldots$ is precisely defined.

in here means is an element of the set of integers from 0 to 1000.

In is written in mathematics as this symbol.

Dot-dot-dot is informal math. We want to write this whole formula in a precisely defined language.

The set of integers from 0 to 1000 is written in TLA+ like this.

Where the operator dot-dot is precisely defined.
if $pc = \text{"start"}$
   then $i' \in 0..1000$
       $pc' = \text{"middle"}$
   else if $pc = \text{"middle"}$
      then $i' = i + 1$
          $pc' = \text{"done"}$
   else no next values

This **then** clause consists of two separate formulas.
if \( pc = \text{"start"} \) 

then \( i' \in 0..1000 \) 
\[ \text{pc}' = \text{"middle"} \]

else if \( pc = \text{"middle"} \) 

then \( i' = i + 1 \) 
\[ \text{pc}' = \text{"done"} \]

else no next values

This \textbf{then} clause is two formulas.

This \textbf{then} clause consists of two separate formulas.
if $pc = \text{"start"}$
  then $i' \in 0..1000$
  $pc' = \text{"middle"}$
else
  if $pc = \text{"middle"}$
    then $i' = i + 1$
    $pc' = \text{"done"}$
  else no next values

It should be a single formula

This then clause consists of two separate formulas.

It should be a single formula asserting that both formulas are true.
if $pc = \text{"start"}$
then $i' \in 0..1000$
$p_{c'} = \text{"middle"}$
else if $pc = \text{"middle"}$
then $i' = i + 1$
$p_{c'} = \text{"done"}$
else no next values

It should be a single formula asserting that both formulas are true.

This then clause consists of two separate formulas.

It should be a single formula asserting that both formulas are true.
if $pc = \text{"start"}$

then $i' \in 0..1000$ and

$$pc' = \text{"middle"}$$

else if $pc = \text{"middle"}$

then $i' = i + 1$

$$pc' = \text{"done"}$$

else no next values

There’s an implicit “and” here

This then clause consists of two separate formulas.

It should be a single formula asserting that both formulas are true.

There’s an implicit “and” here, and we know how to write and in math:
if $pc = \text{"start"}$
   then $i' \in 0..1000 \land$
       $pc' = \text{"middle"}$
   else if $pc = \text{"middle"}$
       then $i' = i + 1$
       $pc' = \text{"done"}$
   else no next values

There's an implicit “and” here that we can replace with $\land$.

This \textit{then} clause consists of two separate formulas. It should be a single formula asserting that both formulas are true. There's an implicit “and” here, and we know how to write \textit{and} in math: we replace it with this \textit{conjunction} symbol.
if \( pc = \text{“start”} \)
    then \((i' \in 0..1000) \land (pc' = \text{“middle”})\)
else if \( pc = \text{“middle”} \)
    then \( i' = i + 1 \)
        \( pc' = \text{“done”} \)
else no next values

Let’s make it easier to read.

This \textbf{then} clause consists of two separate formulas.

It should be a single formula asserting that both formulas are true.

There’s an implicit “and” here, and we know how to write \textbf{and} in math: we replace it with this \textit{conjunction} symbol.

Let’s add some parentheses to make it easier to read.
if $pc = \text{"start"}$

then $(i' \in 0..1000) \land (pc' = \text{"middle"})$

else if $pc = \text{"middle"}$

then $i' = i + 1$

$pc' = \text{"done"}$

else no next values

We do the same thing with the second then clause.
if $pc = "start"$

then $(i' \in 0..1000) \land (pc' = "middle")$

else if $pc = "middle"

then $i' = i + 1$

$pc' = "done"

else no next values

We do the same thing here.

We do the same thing with the second then clause.
if \( pc = "start" \)

then \( (i' \in 0..1000) \land (pc' = "middle") \)

else if \( pc = "middle" \)

then \( (i' = i + 1) \land (pc' = "done") \)

else no next values

We do the same thing here.

We do the same thing with the second then clause.
if \( pc = \text{"start"} \)

then \((i' \in 0..1000) \land \(pc' = \text{"middle"}\)\)

else if \( pc = \text{"middle"} \)

then \((i' = i + 1) \land \(pc' = \text{"done"}\)\)

else no next values

What about “no next values”, which certainly isn’t a mathematical formula.

There’s something important you need to understand.
if $pc = \text{"start"}$
  then $(i' \in 0..1000) \land (pc' = \text{"middle"})$
else if $pc = \text{"middle"}$
  then $(i' = i + 1) \land (pc' = \text{"done"})$
else no next values

What about this?

What about “no next values”, which certainly isn’t a mathematical formula.

There’s something important you need to understand.
if $pc$ = “start”
   then $(i' \in 0..1000) \land (pc' = "middle")$
else if $pc$ = “middle”
   then $(i' = i + 1) \land (pc' = "done")$
   else no next values

We’re not writing instructions for computing something.

What about “no next values”, which certainly isn’t a mathematical formula.

There’s something important you need to understand.

We’re not writing instructions for computing something.
if $pc = \text{“start”}$
  then ($i' \in 0..1000$) $\land$
  ($pc' = \text{“middle”}$)
else if $pc = \text{“middle”}$
  then ($i' = i + 1$) $\land$
  ($pc' = \text{“done”}$)
else no next values

We’re not writing instructions for computing something.

We’re writing a formula relating $i$, $pc$, $i'$, and $pc'$.

What about “no next values”, which certainly isn’t a mathematical formula.

There’s something important you need to understand.

We’re not writing instructions for computing something.

We are writing a formula relating the values of $i$, $pc$, $i'$, and $pc'$
\[
\text{if } pc = \text{"start"} \\
\text{then } (i' \in 0..1000) \land (pc' = \text{"middle"}) \\
\text{else if } pc = \text{"middle"} \\
\text{then } (i' = i + 1) \land (pc' = \text{"done"}) \\
\text{else no next values}
\]

It does not mean: if \( pc = \text{"start"} \)

This formula does not mean that if \( pc \) equals \text{"start"}
if $pc = \text{"start"}$

then $(i' \in 0..1000) \land (pc' = \text{"middle"})$

else if $pc = \text{"middle"}$

then $(i' = i + 1) \land (pc' = \text{"done"})$

else no next values

It does not mean: if $pc = \text{"start"}$ do the then part
if $pc = \text{“start”}$

then $(i' \in 0..1000) \land (pc' = \text{“middle”})$

else if $pc = \text{“middle”}$

then $(i' = i + 1) \land (pc' = \text{“done”})$

else no next values

It does not mean: if $pc = \text{“start”}$ do the then part, otherwise do the else part.

This formula does not mean that if $pc$ equals $\text{start}$ then do the then part otherwise do the else part.
if \( pc = "start" \)

then \( (i' \in 0..1000) \land (pc' = "middle") \)

else if \( pc = "middle" \)

then \( (i' = i + 1) \land (pc' = "done") \)

else no next values

It means: if \( pc = "start" \)

This formula does not mean that if \( pc \) equals \textit{start} then \textbf{do} the then part otherwise \textbf{do} the else part.

The formula \textbf{does} mean that if \( pc \) equals \textit{start}
if $pc = \text{"start"}$

then $(i' \in 0..1000) \land (pc' = \text{"middle"})$

else if $pc = \text{"middle"}$

then $(i' = i + 1) \land (pc' = \text{"done"})$

else no next values

It means: if $pc = \text{"start"}$ the formula equals the then formula

This formula does not mean that if $pc$ equals start then do the then part otherwise do the else part. The formula does mean that if $pc$ equals start then the value of the formula equals the value of the then formula
if \( pc = \text{“start”} \)

\[
\text{then } (i' \in 0..1000) \land (pc' = \text{“middle”})
\]

else if \( pc = \text{“middle”} \)

\[
\text{then } (i' = i + 1) \land (pc' = \text{“done”})
\]

else no next values

It means: if \( pc = \text{“start”} \) the formula equals the then formula, otherwise it equals the else formula.

This formula does not mean that if \( pc \) equals \text{start} then do the then part otherwise do the else part. The formula does mean that if \( pc \) equals \text{start} then the value of the formula equals the value of the then formula otherwise its value equals the value of the else formula.
if $pc = "start"

then $(i' \in 0..1000) \land (pc' = "middle")$

else if $pc = "middle"

then $(i' = i + 1) \land (pc' = "done")$

else no next values

The value of the formula equals true for these values of $i$, $pc$, $i'$, and $pc'$ because:
if \( pc = \text{"start"} \)
\[
\text{then } (i' \in 0..1000) \land (pc' = \text{"middle"})
\]
else if \( pc = \text{"middle"} \)
\[
\text{then } (i' = i + 1) \land (pc' = \text{"done"})
\]
else no next values

The formula equals \textbf{true} for these values:

\[
i = 17 \quad pc = \text{"start"} \quad i' = 534 \quad pc' = \text{"middle"}
\]

The value of the formula equals \textbf{true} for these values of \( i, pc, i', \) and \( pc' \) because:
if \( pc = \text{"start"} \)
\[
\text{then } (i' \in 0..1000) \land
\quad (pc' = \text{"middle"})
\]
else if \( pc = \text{"middle"} \)
\[
\text{then } (i' = i + 1) \land
\quad (pc' = \text{"done"})
\]
else no next values

The formula equals \textbf{true} for these values:
\[
i = 17 \quad pc = \text{"start"} \quad i' = 534 \quad pc' = \text{"middle"}
\]

The value of the formula equals \textbf{true} for these values of \( i, pc, i', \) and \( pc' \) because:

The \textbf{if} test equals \textbf{true}
if \( pc = "start" \)
then \((i' \in 0..1000) \land (pc' = "middle")\)
else if \( pc = "middle" \)
then \((i' = i + 1) \land (pc' = "done")\)
else no next values

The formula equals true for these values:
\[
i = 17 \quad pc = "start" \quad i' = 534 \quad pc' = "middle"
\]

The value of the formula equals true for these values of \( i, pc, i', \) and \( pc' \) because:
The if test equals true because \( pc \) equals start
if \( pc = "start" \)
  then \((i' \in 0..1000) \land (pc' = "middle")\)
else if \( pc = "middle" \)
  then \((i' = i + 1) \land (pc' = "done")\)
  else no next values

The formula equals \textbf{true} for these values:

\[ i = 17 \quad pc = "start" \quad i' = 534 \quad pc' = "middle" \]

The value of the formula equals \textbf{true} for these values of \( i, pc, i', \) and \( pc' \) because:

The \textbf{if} test equals \textbf{true} because \( pc \) equals \textbf{start}

So the value of the formula equals the value of the \textbf{then} clause
if \( pc = \text{"start"} \)

then \((i' \in 0..1000) \land (pc' = \text{"middle"})\)

else if \( pc = \text{"middle"} \)

then \((i' = i + 1) \land (pc' = \text{"done"})\)

else no next values

The formula equals true for these values:

\[ i = 17 \quad pc = \text{"start"} \quad i' = 534 \quad pc' = \text{"middle"} \]

The value of the formula equals true for these values of \( i, pc, i', \) and \( pc' \) because:

The if test equals true because \( pc \) equals start

So the value of the formula equals the value of the then clause

This clause equals true if and only these two formulas equals true.
\[
\begin{align*}
\text{if } pc = \text{"start"} & \quad \text{then } (i' \in 0\ldots1000) \land (pc' = \text{"middle"}) \\
\text{else if } pc = \text{"middle"} & \quad \text{then } (i' = i + 1) \land (pc' = \text{"done"}) \\
\text{else} & \quad \text{no next values}
\end{align*}
\]

The formula equals true for these values:

\[
i = 17 \quad pc = \text{"start"} \quad i' = 534 \quad pc' = \text{"middle"}
\]

The first formula equals true because
if \( pc = \text{"start"} \)
then \( (i' \in 0..1000) \land (pc' = \text{"middle"}) \)
else if \( pc = \text{"middle"} \)
then \( (i' = i + 1) \land (pc' = \text{"done"}) \)
else no next values

The formula equals true for these values:

\[
i = 17 \quad pc = \text{"start"} \quad i' = 534 \quad pc' = \text{"middle"}
\]

The first formula equals true because

\( i' \) equals 534, which is an element of the set of integers from 0 to 1000.
if $pc = \text{“start”}$

then $(i' \in 0..1000) \land (pc' = \text{“middle”})$

else if $pc = \text{“middle”}$

then $(i' = i + 1) \land (pc' = \text{“done”})$

else no next values

The formula equals true for these values:

\begin{align*}
i &= 17 & pc &= \text{“start”} & i' &= 534 & pc' &= \text{“middle”} \\
\end{align*}

The first formula equals true because $i'$ equals 534, which is an element of the set of integers from 0 to 1000.

The second formula equals true because $pc'$ equals middle.
if \( pc = \text{"start"} \)

then \( (i' \in 0\ldots1000) \land (pc' = \text{"middle"}) \)

else if \( pc = \text{"middle"} \)

then \( (i' = i + 1) \land (pc' = \text{"done"}) \)

else no next values

The formula equals **true** for these values:

\[ i = 17 \quad pc = \text{"start"} \quad i' = 534 \quad pc' = \text{"middle"} \]

The first formula equals true because \( i' \) equals 534, which is an element of the set of integers from 0 to 1000.

The second formula equals true because \( pc' \) equals middle.

So the whole formula equals true for these values.
if $pc = \text{"start"}$

then $(i' \in 0..1000) \land (pc' = \text{"middle"})$

else if $pc = \text{"middle"}$

then $(i' = i + 1) \land (pc' = \text{"done"})$

else no next values

The formula equals false for these values because

[slide 96]
if \( pc = "\text{start}" \)

then \((i' \in 0..1000) \land (pc' = "\text{middle}")\)

else if \( pc = "\text{middle}" \)

then \((i' = i + 1) \land (pc' = "\text{done}")\)

else no next values

The formula equals false for these values:

\( i = 534 \quad pc = "\text{middle}" \quad i' = 77 \quad pc' = "\text{done}" \)

The formula equals false for these values because
if \( pc = \text{"start"} \) \\
then \((i' \in 0..1000) \land (pc' = \text{"middle"})\) \\
else if \( pc = \text{"middle"} \) \\
then \((i' = i + 1) \land (pc' = \text{"done"})\) \\
else no next values

The formula equals **false** for these values:

\( i = 534 \quad pc = \text{"middle"} \quad i' = 77 \quad pc' = \text{"done"} \)

The formula equals false for these values because

The **if** test equals false
\[
\text{if } pc = \text{“start”}
\]
\[
\text{then } (i' \in 0..1000) \land
(\text{pc}' = \text{“middle”})
\]
\[
\text{else if } pc = \text{“middle”}
\]
\[
\text{then } (i' = i + 1) \land
(\text{pc}' = \text{“done”})
\]
\[
\text{else no next values}
\]

The formula equals \textbf{false} for these values:
\[
i = 534 \quad pc = \text{“middle”} \quad i' = 77 \quad pc' = \text{“done”}
\]

The formula equals false for these values because

The \textbf{if} test equals false
so the value of the formula equals the value of the \textbf{else} clause.

That clause is an \textbf{if} formula

[slide 99]
if $pc$ = “start”
\[
\text{then } (i' \in 0..1000) \land (pc' = “middle”)
\]
else if $pc$ = “middle”
\[
\text{then } (i' = i + 1) \land (pc' = “done”)
\]
else no next values

The formula equals **false** for these values:
\[
i = 534 \quad pc = “middle” \quad i' = 77 \quad pc' = “done”
\]

The formula equals false for these values because

The **if** test equals false
so the value of the formula equals the value of the **else** clause.

That clause is an **if** formula  whose whose test equals true,
if $pc = "start"
  then $(i' \in 0..1000) \land (pc' = "middle")$
else if $pc = "middle"
  then $(i' = i + 1) \land (pc' = "done")$
else no next values

The formula equals **false** for these values:

\[ i = 534 \quad pc = "middle" \quad i' = 77 \quad pc' = "done" \]

The formula equals false for these values because

The **if** test equals false
so the value of the formula equals the value of the **else** clause.

That clause is an **if** formula whose whose test equals true,
so it equals its **then** clause.

The value of that clause equals true if and only if
if $pc = \text{“start”}$

then $(i' \in 0..1000) \land (pc' = \text{“middle”})$

else if $pc = \text{“middle”}$

then $(i' = i + 1) \land (pc' = \text{“done”})$

else no next values

The formula equals \textbf{false} for these values:

$i = 534 \quad pc = \text{“middle”} \quad i' = 77 \quad pc' = \text{“done”}$

these two formulas both equal true.
if \( pc = "start" \)
\[
\text{then } (i' \in 0..1000) \land \\
(pc' = "middle")
\]
else if \( pc = "middle" \)
\[
\text{then } (i' = i + 1) \land \\
(pc' = "done")
\]
else no next values

The formula equals \textbf{false} for these values:
\[
i = 534 \quad pc = "middle" \quad i' = 77 \quad pc' = "done"
\]

these two formulas both equal true.

But this formula equals false
\[
\text{if } pc = \text{"start"} \\
\quad \text{then } (i' \in 0..1000) \land (pc' = \text{"middle"}) \\
\quad \text{else if } pc = \text{"middle"} \\
\quad \quad \text{then } (i' = i + 1) \land (pc' = \text{"done"}) \\
\quad \quad \text{else no next values}
\]

The formula equals \textbf{false} for these values:

\[
i = 534 \quad pc = \text{"middle"} \quad i' = 77 \quad pc' = \text{"done"}
\]

these two formulas both equal true.

But this formula equals false \textbf{because} \(i'\)
if $pc = \text{"start"}$
  then $(i' \in 0..1000) \land (pc' = \text{"middle"})$
else if $pc = \text{"middle"}$
  then $(i' = i + 1) \land (pc' = \text{"done"})$
else no next values

The formula equals **false** for these values:

\[
i = 534 \quad pc = \text{"middle"} \quad i' = 77 \quad pc' = \text{"done"}
\]

these two formulas both equal true.

But this formula equals false because $i'$ does not equal $i$ plus 1.
if \( pc = \text{“start”} \)

then \( (i' \in 0\ldots1000) \land (pc' = \text{“middle”}) \)

else if \( pc = \text{“middle”} \)

then \( (i' = i + 1) \land (pc' = \text{“done”}) \)

else no next values

The formula equals **false** for these values:

\[ i = 534 \quad pc = \text{“middle”} \quad i' = 77 \quad pc' = \text{“done”} \]

these two formulas both equal true.

But this formula equals false because \( i' \) does not equal \( i \) plus 1.

So the entire formula equals false.
if $pc = "start"
  then $(i' \in 0..1000) \land (pc' = "middle")$
else if $pc = "middle"
  then $(i' = i + 1) \land (pc' = "done")$
else no next values

Now let’s return to the no next values clause.
if $pc = "start"
    then $(i' \in 0\ldots1000) \land
       (pc' = "middle")$
else if $pc = "middle"
    then $(i' = i + 1) \land
       (pc' = "done")$
else no next values

Let's return to this clause.

Now let's return to the no next values clause.
if $pc = \text{"start"}$
then $(i' \in 0\ldots1000) \land (pc' = \text{"middle"})$
else if $pc = \text{"middle"}$
then $(i' = i + 1) \land (pc' = \text{"done"})$
else no next values

It should be a formula that does not equal true for any values of $i$, $pc$, $i'$, and $pc'$.

Now let’s return to the no next values clause.

This clause should be a formula that does not equal true for any values of $i$, $pc$, $i'$, and $pc'$. 
If \( pc = \text{"start"} \)
\[
\text{then } (i' \in 0..1000) \land (pc' = \text{"middle"})
\]
else if \( pc = \text{"middle"} \)
\[
\text{then } (i' = i + 1) \land (pc' = \text{"done"})
\]
else no next values

The simplest
It should be a formula that does not equal true
for any values of \( i, pc, i', \) and \( pc' \).

Now let's return to the *no next values* clause.

This clause should be a formula that does not equal true for *any* values of \( i, pc, i', \) and \( pc' \).

Let's use the simplest such formula, which is one that always equals false—namely,
if \( pc = \text{“start”} \)

then \((i' \in 0\ldots1000) \land (pc' = \text{“middle”})\)

else if \( pc = \text{“middle”} \)

then \((i' = i + 1) \land (pc' = \text{“done”})\)

else FALSE

The simplest

It should be a formula that does not equal true

for any values of \( i, pc, i', \) and \( pc' \).

Now let’s return to the no next values clause.

This clause should be a formula that does not equal true for any values of \( i, pc, i', \) and \( pc' \).

Let’s use the simplest such formula, which is one that always equals false—namely, the formula false.
if \( pc = \text{"start"} \)

then \( i' \in 0\ldots1000 \) \( \land \)

\( (pc' = \text{"middle"}) \)

else if \( pc = \text{"middle"} \)

then \( i' = i + 1 \) \( \land \)

\( (pc' = \text{"done"}) \)

else FALSE

In TLA+ most keywords
if $pc = \text{"start"}$
then $(i' \in 0..1000) \land (pc' = \text{"middle"})$
else if $pc = \text{"middle"}$
then $(i' = i + 1) \land (pc' = \text{"done"})$
else FALSE

In TLA+ most keywords
IF $pc = \text{"start"}$
  THEN $(i' \in 0..1000) \land (pc' = \text{"middle"})$
ELSE IF $pc = \text{"middle"}$
  THEN $(i' = i + 1) \land (pc' = \text{"done"})$
ELSE FALSE

In TLA$^+$ most keywords are in uppercase.

In TLA+ most keywords are in uppercase letters.
IF $pc = \text{“start”}$

THEN $(i' \in 0..1000) \land$

$(pc' = \text{“middle”})$

ELSE IF $pc = \text{“middle”}$

THEN $(i' = i + 1) \land$

$(pc' = \text{“done”})$

ELSE FALSE

This is a TLA$^+$ formula.

In TLA$^+$ most keywords are in uppercase letters.

This is now a TLA$^+$ formula.
IF \( pc = "\text{start}" \)
    THEN \( (i' \in 0..1000) \land (pc' = "\text{middle}")) \)
ELSE IF \( pc = "\text{middle}" \)
    THEN \( (i' = i + 1) \land (pc' = "\text{done}")) \)
ELSE FALSE

This is a pretty-printed TLA+ formula.

In TLA+ most keywords are in uppercase letters.

This is now a TLA+ formula. That is, a pretty-printed TLA+ formula.
If $pc = \text{"start"}$
   THEN $(i' \in 0..1000) \land$
       $(pc' = \text{"middle"})$
   ELSE IF $pc = \text{"middle"}$
       THEN $(i' = i + 1) \land$
            $(pc' = \text{"done"})$
   ELSE FALSE

The TLA+ source is in ASCII

In TLA+ most keywords are in uppercase letters.

This is now a TLA+ formula. That is, a pretty-printed TLA+ formula.

The TLA+ source is in ASCII,
IF \( pc = \text{“start”} \)
THEN \((i' \in 0..1000) \land (pc' = \text{“middle”})\)
ELSE IF \( pc = \text{“middle”} \)
THEN \((i' = i + 1) \land (pc' = \text{“done”})\)
ELSE FALSE

The TLA\(^+\) source is in ASCII, with \(\land\) typed as \(\\backslash\slash\)
IF \( pc = \text{"start"} \)
\[
\text{THEN } (i' \in 0..1000) \land \\
(pc' = \text{"middle"})
\]
ELSE IF \( pc = \text{"middle"} \)
\[
\text{THEN } (i' = i + 1) \land \\
(pc' = \text{"done"})
\]
ELSE FALSE

The TLA\(^+\) source is in ASCII, with \( \land \) typed as \( \backslash \land \) and \( \in \) typed as \( \backslash \text{in} \).

In TLA\(^+\) most keywords are in uppercase letters.

This is now a TLA\(^+\) formula. That is, a pretty-printed TLA\(^+\) formula.

The TLA\(^+\) source is in ASCII, with \textit{and} typed as forward-slash backslash and the \textit{element-of} symbol typed like this.

[slide 119]
IF pc = "start"
    THEN (i' \in 0..1000) /\
        (pc' = "middle")
ELSE IF pc = "middle"
    THEN (i' = i+1) /\
        (pc' = "done")
ELSE FALSE

This is what it looks like in ASCII.
IF $pc = \text{"start"}$

THEN $(i' \in 0..1000) \land$

$(pc' = \text{"middle"})$

ELSE IF $pc = \text{"middle"}$

THEN $(i' = i + 1) \land$

$(pc' = \text{"done"})$

ELSE FALSE

This version is easier for most people to read.

This version is easier for most people to read.
IF $pc = \text{"start"}$

THEN $(i' \in 0..1000) \land (pc' = \text{"middle\"})$

ELSE IF $pc = \text{"middle\"}$

THEN $(i' = i + 1) \land (pc' = \text{"done\")}$

ELSE FALSE

This version is easier for most people to read.

I’ll use it for now.
The Complete Mathematical Description

We have now written a complete mathematical description of the program as two formulas.
The Complete Mathematical Description

Initial-state formula: \((i = 0) \land (pc = "start")\)

We have now written a complete mathematical description of the program as two formulas.

The initial-state formula.

[slide 124]
The Complete Mathematical Description

Initial-state formula: \((i = 0) \land (pc = "start")\)

Next-state formula: IF \(pc = "start"\)
THEN \((i' \in 0..1000) \land (pc' = "middle")\)
ELSE IF \(pc = "middle"\)
THEN \((i' = i + 1) \land (pc' = "done")\)
ELSE FALSE

We have now written a complete mathematical description of the program as two formulas.

The initial-state formula. and the next-state formula.
The Complete Mathematical Description

Initial-state formula: \((i = 0) \land (pc = "start")\)

Next-state formula: IF \(pc = "start"\)
THEN \((i' \in 0..1000) \land (pc' = "middle")\)
ELSE IF \(pc = "middle"\)
THEN \((i' = i + 1) \land (pc' = "done")\)
ELSE FALSE

There's a nicer way to write this.

We have now written a complete mathematical description of the program as two formulas.

The initial-state formula. and the next-state formula.

But there's a nicer way to write the next-state formula.
A NICER WAY TO WRITE THE NEXT-STATE FORMULA

Let’s now see how.
IF \( pc = “start” \)
\[
\text{THEN } (i' \in 0..1000) \land
   (pc' = “middle”)
\]
ELSE IF \( pc = “middle” \)
\[
\text{THEN } (i' = i + 1) \land
   (pc' = “done”)
\]
ELSE FALSE

I’ll start by hiding some of the details.
IF  $pc = "start"$
THEN  $\left( i' \in 0..1000 \right) \land (pc' = "middle")$
ELSE  IF $pc = "middle"$
THEN  $\left( i' = i + 1 \right) \land (pc' = "done")$
ELSE  FALSE

Let’s call these two formulas
IF $pc = \text{“start”}$
  THEN $A$
ELSE IF $pc = \text{“middle”}$
  THEN $(i' = i + 1) \land (pc' = \text{“done”})$
  ELSE FALSE

Let’s call these two formulas $A$

I’ll start by hiding some of the details.

Let’s call these two formulas $A$
IF \( pc = \text{“start”} \) THEN

ELSE IF \( pc = \text{“middle”} \) THEN

ELSE FALSE

Let’s call these two formulas \( A \) and \( B \).

I’ll start by hiding some of the details.

Let’s call these two formulas \( A \) and \( B \).
IF $pc = \text{"start"}$
   THEN $A$
ELSE IF $pc = \text{"middle"}$
   THEN $B$
ELSE FALSE
IF \( pc = "start" \)
    THEN \( A \)
ELSE IF \( pc = "middle" \)
    THEN \( B \)
    ELSE FALSE

There are two cases when the formula is true:

1. \( pc \) equals "start" and \( A \) are both true.
2. \( pc \) equals "middle" and \( B \) are both true.

In other words, the single formula
\( pc \) equals "start" and \( A \)
is true.

Case 2:
\( pc \) equals "middle" and \( B \)
is true.

[slide 133]
IF $pc$ = "start"
   THEN $A$
ELSE IF $pc$ = "middle"
   THEN $B$
   ELSE FALSE

There are two cases when the formula is true:

There are two cases when the formula is true:
IF \( pc = "start" \) THEN \( A \)
ELSE IF \( pc = "middle" \) THEN \( B \) ELSE FALSE

There are two cases when the formula is true:

1. \( pc = "start" \) and \( A \) are true.

There are two cases when the formula is true:

Case 1: \( pc \) equals \( start \) and \( A \) are both true.
IF \( pc = \text{"start"} \)
THEN \( A \)
ELSE IF \( pc = \text{"middle"} \)
THEN \( B \)
ELSE FALSE

There are two cases when the formula is true: 

1. \((pc = \text{"start"}) \land A\) is true.

There are two cases when the formula is true:

Case 1: \( pc \) equals \textit{start} and \( A \) are both true.

In other words, the single formula \( pc \) equals \textit{start} and \( A \) is true.
IF $pc = "start"$
THEN $A$
ELSE IF $pc = "middle"$
THEN $B$
ELSE FALSE

There are two cases when the formula is true:

1. $(pc = "start") \land A$ is true.
2. $pc = "middle"$ and $B$ are both true.

There are two cases when the formula is true:

Case 1: $pc$ equals $start$ and $A$ are both true.
In other words, the single formula $pc$ equals $start$ and $A$ is true.

Case 2: $pc$ equals $middle$ and $B$ are both true.
IF \( pc = \text{"start"} \)
    THEN \( A \)
ELSE IF \( pc = \text{"middle"} \)
    THEN \( B \)
ELSE FALSE

There are two cases when the formula is true:

1. \((pc = \text{"start"}) \land A\) is true.
2. \((pc = \text{"middle"}) \land B\) is true.

There are two cases when the formula is true:

Case 1: \( pc \) equals \( \text{start} \) and \( A \) are both true.
In other words, the single formula \( pc \) equals \( \text{start} \) and \( A \) is true.

Case 2: \( pc \) equals \( \text{middle} \) and \( B \) are both true.
In other words, the single formula \( pc \) equals \( \text{middle} \) and \( B \) is true.
IF \( pc = "start" \) THEN \( A \)  
ELSE IF \( pc = "middle" \) THEN \( B \)  
ELSE FALSE

There are two cases when the formula is true:

1. \(( pc = "start" ) \land A \) is true.
2. \(( pc = "middle" ) \land B \) is true.

So we can rewrite the formula like this.
To turn it into a mathematical formula,
IF $pc = "start"$
  THEN $A$
ELSE IF $pc = "middle"
  THEN $B$
ELSE FALSE

$(pc = "start") \land A$
or $(pc = "middle") \land B$

Must replace “or” by a mathematical operator.

So we can rewrite the formula like this.
To turn it into a mathematical formula,
we must replace the word *or* by a mathematical operator.
IF \( pc = "start" \)
   THEN \( A \)
ELSE IF \( pc = "middle" \)
   THEN \( B \)
ELSE FALSE

\[(pc = "start") \land A \]
\( \lor \)
\[(pc = "middle") \land B \]

Must replace “\( \lor \)” by a mathematical operator.

Written \(||\) in some programming languages.

So we can rewrite the formula like this.
To turn it into a mathematical formula, we must replace the word \( \lor \) by a mathematical operator.

That operator is written \( \text{bar bar} \) in some programming languages.
IF \( pc = "start" \)
THEN \( A \)
ELSE IF \( pc = "middle" \)
THEN \( B \)
ELSE FALSE

\[ ((pc = "start") \land A) \lor ((pc = "middle") \land B) \]

Must replace “or” by a mathematical operator.

Written \( \lor \) in some programming languages.

Written \( \lor \) in mathematics.

So we can rewrite the formula like this.
To turn it into a mathematical formula, we must replace the word \( or \) by a mathematical operator.

That operator is written bar bar bar in some programming languages.

It’s written as this symbol in mathematics.
Now let's replace $A$ and $B$ by their original formulas. First let's give us some more room. We replace $A$. And we replace $B$. And now let's format it a little better.

\[
((pc = \text{"start"}) \land A) \\
\lor ((pc = \text{"middle"}) \land B)
\]
$((pc = \text{"start"}) \land A) \lor ((pc = \text{"middle"}) \land B)$

Let’s replace $A$ and $B$ by their original formulas.

Now let’s replace $A$ and $B$ by their original formulas.
Now let’s replace $A$ and $B$ by their original formulas.

First let’s give us some more room.
Now let’s replace $A$ and $B$ by their original formulas.

First let’s give us some more room.

We replace $A$. 

\[( (pc = \text{"start"}) \land (i' \in 0..1000) \land (pc' = \text{"middle"})) \lor ((pc = \text{"middle"}) \land B) \]
Now let’s replace \( A \) and \( B \) by their original formulas.

First let’s give us some more room.

We replace \( A \).

And we replace \( B \).
\[
((pc = "start") \land \\
(i' \in 0..1000) \land \\
(pc' = "middle")) \\
\lor \\
((pc = "middle") \land \\
(i' = i + 1) \land \\
(pc' = "done"))
\]

Let's format it better.

Now let's replace \( A \) and \( B \) by their original formulas.

First let's give us some more room.

We replace \( A \).

And we replace \( B \).

And now let's format it a little better.
\[
\left( (pc = "start") \right) \\
\land (i' \in 0 \ldots 1000) \\
\land (pc' = "middle") \\
\lor \left( (pc = "middle") \\
\land (i' = i + 1) \\
\land (pc' = "done") \right)
\]

These parentheses aren't necessary and with this formatting they don't help.

[slide 149]
\[
(\ (pc = "start")
\land (i' \in 0..1000)
\land (pc' = "middle")\) \\
\lor (\ (pc = "middle")
\land (i' = i + 1)
\land (pc' = "done")\)
\]

These parentheses aren’t needed and don’t help
\( ( \ pc = \ "\text{start}\" \\
\& \ i' \in 0..1000 \\
\& \ pc' = \ "\text{middle}\" ) \)

\( \lor \ ( \ pc = \ "\text{middle}\" \\
\& \ i' = i + 1 \\
\& \ pc' = \ "\text{done}\" ) \)

So let’s remove them.
\[
\begin{align*}
&\left( pc = \text{"start"} \\
&\quad \land i' \in 0..1000 \\
&\quad \land pc' = \text{"middle"} \right] \\
\lor \left( pc = \text{"middle"} \\
&\quad \land i' = i + 1 \\
&\quad \land pc' = \text{"done"} \right)
\end{align*}
\]

Widely separated matching parentheses make formulas hard to read. (They’re not very far apart here, but they could be in a larger formula.)
\[ \begin{align*}
&\land pc = "start" \\
&\land i' \in 0..1000 \\
&\land pc' = "middle" \\
&\lor (pc = "middle" \\
&\land i' = i + 1 \\
&\land pc' = "done")
\end{align*} \]

Widely separated matching parentheses make formulas hard to read. (They’re not very far apart here, but they could be in a larger formula.) TLA+ lets us eliminate them by adding this extra \textit{and} symbol.
Widely separated matching parentheses make formulas hard to read.
(They’re not very far apart here, but they could be in a larger formula.)

TLA+ lets us eliminate them by adding this extra \textit{and} symbol.

This turns the subformula into a bulleted \textit{and} list that is ended by
\[
\wedge pc = "start"
\wedge i' \in 0..1000
\wedge pc' = "middle"
\]

Widely separated matching parentheses make formulas hard to read. (They’re not very far apart here, but they could be in a larger formula.) TLA+ lets us eliminate them by adding this extra \textit{and} symbol. This turns the subformula into a bulleted \textit{and} list that is ended by any following token to the left of the \textit{and} symbols.

[slide 155]
As if these parentheses were there.
\[ \lor ( pc = \text{“middle”} \land i’ = i + 1 \land pc’ = \text{“done”} ) \]

As if these parentheses were there.

Let’s do the same thing with this subformula.
\[ \lor \land pc = \text"middle" \land i' = i + 1 \land pc' = \text"done" \]

As if these parentheses were there.

Let’s do the same thing with this subformula.
\[ \land \ pc = \text{"start"} \]
\[ \land \ i' \in 0..1000 \]
\[ \land \ pc' = \text{"middle"} \]
\[ \lor \ \land \ pc = \text{"middle"} \]
\[ \land \ i' = i + 1 \]
\[ \land \ pc' = \text{"done"} \]

Let’s do the same thing
Let's compare the TLA+ formula with the corresponding C code.

Let's do the same thing for the or.

\[
\begin{align*}
\land pc = \text{"start"} \\
\land i' \in 0..1000 \\
\land pc' = \text{"middle"} \\
\lor \land pc = \text{"middle"} \\
\land i' = i + 1 \\
\land pc' = \text{"done"}
\end{align*}
\]
\[ \begin{align*}
\lor & \quad \land pc = "start" \\
\land & \quad i' \in 0..1000 \\
\land & \quad pc' = "middle" \\
\lor & \quad \land pc = "middle" \\
\land & \quad i' = i + 1 \\
\land & \quad pc' = "done"
\end{align*} \]

Let's do the same thing for the or.

TLA+ also allows bulleted or lists.
(\forall \ \wedge \ pc = \text{“start”} \\
\wedge \ i' \in 0..1000 \\
\wedge \ pc' = \text{“middle”} \\
\forall \ \wedge \ pc = \text{“middle”} \\
\wedge \ i' = i + 1 \\
\wedge \ pc' = \text{“done”} )

Let’s do the same thing for the \textit{or}.

TLA+ also allows bulleted \textit{or} lists.

There are implicit parentheses around the formula.
\[ \forall \wedge pc = \text{“start”} \]
\[ \wedge i' \in 0..1000 \]
\[ \wedge pc' = \text{“middle”} \]
\[ \forall \wedge pc = \text{“middle”} \]
\[ \wedge i' = i + 1 \]
\[ \wedge pc' = \text{“done”} \]

Let’s do the same thing for the or.

TLA+ also allows bulleted or lists.

There are implicit parentheses around the formula.
\[
\begin{align*}
&\forall \land pc = "start" \\
&\land i' \in 0..1000 \\
&\land pc' = "middle" \\
&\forall \land pc = "middle" \\
&\land i' = i + 1 \\
&\land pc' = "done"
\end{align*}
\]

Let’s compare the TLA+ formula with the corresponding C code.

Let’s do the same thing for the or.

TLA+ also allows bulleted or lists.

There are implicit parentheses around the formula.

Now let’s compare the TLA+ formula with the corresponding C code, which...
is the C code without the declaration of $i$. 
The C code likely seems simpler because it’s more familiar. This is the C code without the declaration of \( i \).

The C code probably seems simpler than the TLA+ formula because it’s more familiar to you.
int i;
void main()
{
    i = someNumber();
    i = i + 1;
}

\[ \forall \land pc = \text{"start"} \land i' \in 0..1000 \land pc' = \text{"middle"} \land pc = \text{"middle"} \land i' = i + 1 \land pc' = \text{"done"} \]

The C code probably seems simpler because it’s more familiar.

But it isn’t really simpler.

is the C code without the declaration of i.

The C code probably seems simpler than the TLA+ formula because it’s more familiar to you.

But the C code isn’t really simpler.
int i;
void main()
{ i = someNumber();
  i = i + 1;
}

\[ \sqrt{\land pc = "start" \land i' \in 0..1000 \land pc' = "middle" \land pc = "middle" \land i' = i + 1 \land pc' = "done" } \]

= in TLA\(^+\) means equality, as in \(2 + 2 = 4\).

is the C code without the declaration of \(i\).

The C code probably seems simpler than the TLA\(^+\) formula because it's more familiar to you.

But the C code isn't really simpler.

For one thing, the equal sign in TLA\(^+\) means equality, just as in grammar school, when you wrote two plus two equals 4.

[slide 168]
int i;
void main()
{
 i = someNumber();
 i = i + 1;
}

\[ \text{in TLA}^+ \text{ means equality, as in } 2 + 2 = 4 . \]

\[ \text{in C means assignment, which isn't so simple.} \]
The equals sign in C means assignment, which isn’t so simple.

But the big difference between math and C is that math is much, much more expressive.
int i;

void main()
{
    i = someNumber();
    i = i + 1;
}

What about *someNumber*?

The equals sign in C means assignment, which isn’t so simple.

But the big difference between math and C is that math is much, much more expressive.

What about *someNumber*?
int i;
void main()
{
  { i = someNumber();
    i = i + 1;
  }

\[
\begin{align*}
\forall \land pc &= \text{“start”} \\
\land i' &\in 0..1000 \\
\land pc' &= \text{“middle”} \\
\forall \land pc &= \text{“middle”} \\
\land i' &= i + 1 \\
\land pc' &= \text{“done”}
\end{align*}
\]

Its execution is nondeterministic.

The equals sign in C means assignment, which isn’t so simple.

But the big difference between math and C is that math is much, much more expressive.

What about someNumber?

Its execution is nondeterministic.
We need nondeterminism to describe systems,

\[
\begin{align*}
\text{int } i; & \quad \forall \land \quad pc = \text{“start”} \\
\text{void main()} & \quad \land \quad i' \in 0..1000 \\
\{ & \quad \land \quad pc' = \text{“middle”} \\
\quad i = \text{someNumber}(); & \quad \forall \land \quad pc = \text{“middle”} \\
\quad i = i + 1; & \quad \land \quad i' = i + 1 \\
\} & \quad \land \quad pc' = \text{“done”}
\end{align*}
\]

We need nondeterminism like this to describe systems,
int i;

void main()
{
    i = someNumber();
    i = i + 1;
}

\[ \forall \land pc = \text{“start”} \land i' \in 0..1000 \land pc' = \text{“middle”} \land \exists pc = \text{“middle”} \land i' = i + 1 \land pc' = \text{“done”} \]

We need nondeterminism to describe systems, because we can’t predict in what order things happen.
int i;
void main()
{
   i = someNumber();
   i = i + 1;
}

\[ \begin{align*}
   \lor \land pc & = \text{"start"} \\
   \land i' & \in 0..1000 \\
   \land pc' & = \text{"middle"} \\
   \lor \land pc & = \text{"middle"} \\
   \land i' & = i + 1 \\
   \land pc' & = \text{"done"} 
\end{align*} \]

Look how easily it's described in math.

We need nondeterminism like this to describe systems, because we can't predict in what order things happen.

Look how easily nondeterminism is described in math.

[slide 175]
Look how easily it’s described in math.

Programming languages weren’t designed to express nondeterminism.

Commonly used programming languages were not designed to express nondeterminism.
They lack more than constructs for nondeterminism.

Commonly used programming languages were not designed to express nondeterminism.

Programming languages lack much more than constructs for nondeterminism.
int i;
void main()
{
i = someNumber();
i = i + 1;
}

∧ pc = “start”
∧ i' ∈ 0..1000
∧ pc' = “middle”

∨ ∧ pc = “middle”
∧ i' = i + 1
∧ pc' = “done”

They lack more than constructs for nondeterminism.

Programming languages don’t abstract above the code level.

Commonly used programming languages were not designed to express nondeterminism.

Programming languages lack much more than constructs for nondeterminism.

They don’t let you abstract above the code level.
\( \forall \wedge pc = \text{“start”} \)
\( \wedge i' \in 0..1000 \)
\( \wedge pc' = \text{“middle”} \)
\( \forall \wedge pc = \text{“middle”} \)
\( \wedge i' = i + 1 \)
\( \wedge pc' = \text{“done”} \)

It’s important to remember that this is a formula

It’s important to remember that this is a formula,
\( \forall \land \text{pc} = \text{"start"} \)
\( \land \text{i'} \in 0..1000 \)
\( \land \text{pc'} = \text{"middle"} \)
\( \forall \land \text{pc} = \text{"middle"} \)
\( \land \text{i'} = i + 1 \)
\( \land \text{pc'} = \text{"done"} \)

It’s important to remember that this is a formula, not a sequence of commands.
\[ \forall \wedge \ pc = \text{“start”} \\wedge i' \in 0..1000 \wedge pc' = \text{“middle”} \\wedge \forall \wedge pc = \text{“middle”} \wedge i' = i + 1 \wedge pc' = \text{“done”} \]

\[ \forall \text{ is commutative} \]

It's important to remember that this is a formula, not a sequence of commands.

or is commutative
\[\begin{array}{l}
\lor \quad \land \ p c = “\text{start}” \\
\lor \quad \land \ i’ \in 0..1000 \\
\lor \quad \land \ p c’ = “\text{middle}” \\
\lor \quad \land \ p c = “\text{middle}” \\
\lor \quad \land \ i’ = i + 1 \\
\lor \quad \land \ p c’ = “\text{done}”
\end{array}\]

\[\lor\] is commutative, so interchanging these sub-formulas

It’s important to remember that this is a formula, not a sequence of commands.

or is commutative so interchanging these sub-formulas
\[ \forall \quad \land \quad pc = \text{"start"} \\
\land \quad i' \in 0..1000 \\
\land \quad pc' = \text{"middle"} \]

\[ \lor \quad \land \quad pc = \text{"middle"} \\
\land \quad i' = i + 1 \\
\land \quad pc' = \text{"done"} \]

\[ \lor \quad \land \quad pc = \text{"middle"} \\
\land \quad i' = i + 1 \\
\land \quad pc' = \text{"done"} \]

\[ \lor \quad \land \quad pc = \text{"start"} \\
\land \quad i' \in 0..1000 \\
\land \quad pc' = \text{"middle"} \]

\[ \land \quad pc = \text{"middle"} \\
\land \quad i' = i + 1 \\
\land \quad pc' = \text{"done"} \]

\[ \land \quad pc = \text{"start"} \\
\land \quad i' \in 0..1000 \\
\land \quad pc' = \text{"middle"} \]

\[ \forall \quad \text{is commutative, so interchanging these sub-formulas yields an equivalent formula.} \]

It's important to remember that this is a formula, not a sequence of commands.

\textit{or} is commutative so interchanging these sub-formulas yields an equivalent formula.
\[ \begin{align*}
\lor & \land pc = \text{“start”} \\
& \land i' \in 0..1000 \\
& \land pc' = \text{“middle”} \\
\lor & \land pc = \text{“middle”} \\
& \land i' = i + 1 \\
& \land pc' = \text{“done”} \\
\lor & \land pc = \text{“middle”} \\
& \land i' = i + 1 \\
& \land pc' = \text{“done”} \\
\lor & \land pc = \text{“start”} \\
& \land i' \in 0..1000 \\
& \land pc' = \text{“middle”} \\
\end{align*} \]

\[ \lor \text{ is commutative, so interchanging these sub-formulas yields an equivalent formula.} \]

It’s important to remember that this is a formula, not a sequence of commands.

or is commutative so interchanging these sub-formulas yields an equivalent formula.
\( \square \land p_c = "start" \)
\( \land i' \in 0 \ldots 1000 \)
\( \land p'_c = "middle" \)
\( \land p_c = "middle" \)
\( \land i' = i + 1 \)
\( \land p'_c = "done" \)

\( \land \) is also commutative

It's important to remember that this is a formula, not a sequence of commands.

\textit{or} is commutative, so interchanging these sub-formulas yields an equivalent formula.

\textit{and} is also commutative
\[ \forall \ \land \ pc = \text{"start"} \land i' \in 0..1000 \land pc' = \text{"middle"} \]
\[ \forall \ \land \ pc = \text{"middle"} \land i' = i + 1 \land pc' = \text{"done"} \]

\[ \land \ \text{is also commutative, so interchanging these sub-formulas} \]

It’s important to remember that this is a formula, not a sequence of commands.

or is commutative so interchanging these sub-formulas yields an equivalent formula.

and is also commutative so interchanging these sub-formulas
∧ is also commutative, so interchanging these sub-formulas also yields an equivalent formula.

It’s important to remember that this is a formula, not a sequence of commands.

or is commutative so interchanging these sub-formulas yields an equivalent formula.

and is also commutative so interchanging these sub-formulas also yields an equivalent formula.
\[
\begin{align*}
\forall \land pc &= \text{"start"} \\
\land i' &\in 0..1000 \\
\land pc' &= \text{"middle"} \\
\forall \land pc &= \text{"middle"} \\
\land i' &= i + 1 \\
\land pc' &= \text{"done"} \\
\forall \land pc' &= \text{"middle"} \\
\land pc &= \text{"start"} \\
\land i' &\in 0..1000 \\
\land pc &= \text{"middle"} \\
\land i' &= i + 1 \\
\land pc' &= \text{"done"}
\end{align*}
\]

\[
\land \text{is also commutative, so interchanging these sub-formulas also yields an equivalent formula.}
\]

It's important to remember that this is a formula, not a sequence of commands.

*or* is commutative so interchanging these sub-formulas yields an equivalent formula.

*and* is also commutative so interchanging these sub-formulas also yields an equivalent formula.
\[ \lor \land pc = \text{“start”} \\
\land i' \in 0..1000 \\
\land pc' = \text{“middle”} \\
\lor \land pc = \text{“middle”} \\
\land i' = i + 1 \\
\land pc' = \text{“done”} \]

It’s important to remember that this is a formula, not a sequence of commands.

\textit{or} is commutative so interchanging these sub-formulas yields an equivalent formula.

\textit{and} is also commutative so interchanging these sub-formulas also yields an equivalent formula.
THE COMPLETE TLA+ SPEC

The complete TLA+ Specification.
The Complete Spec in Math

Initial-state formula: \((i = 0) \land (pc = \text{“start”})\)

Next-state formula: \(\lor \land pc = \text{“start”} \land i' \in 0\ldots1000 \land pc' = \text{“middle”} \lor \land pc = \text{“middle”} \land i' = i + 1 \land pc' = \text{“done”}\)

This is the complete specification in mathematics.
The Complete Spec in Math

Initial-state formula: \((i = 0) \land (pc = \text{“start”})\)

This is the complete specification in mathematics.

The initial-state formula can also be written like this.
The Complete Spec in Math

Initial-state formula: \[ \land \ i = 0 \land pc = "start" \]

This is the complete specification in mathematics.

The initial-state formula can also be written like this.

But this
The Complete Spec in Math

Initial-state formula: \((i = 0) \land (pc = \text{"start"})\)

This is the complete specification in mathematics.
The initial-state formula can also be written like this.
But this takes less space.
The Complete Spec in Math

Initial-state formula: \((i = 0) \land (pc = \text{“start”})\)

Next-state formula: \(\lor \land pc = \text{“start”}\)
\(\land i' \in 0..1000\)
\(\land pc' = \text{“middle”}\)
\(\lor \land pc = \text{“middle”}\)
\(\land i' = i + 1\)
\(\land pc' = \text{“done”}\)

A TLA+ specification has some additional stuff.
A TLA+ spec appears in a module.
A TLA\(^+\) spec appears in a module.
This module is named *SimpleProgram*.

A TLA*spec appears in a module.

This module is named *SimpleProgram*.

[slide 198]
A TLA+ spec appears in a module.

This module is named *SimpleProgram*.

This **EXTENDS** statement
A TLA+ spec appears in a module.

This module is named *SimpleProgram*.

This `EXTENDS` statement imports arithmetic operators like plus and dot-dot.
Identifiers must be defined or declared before they’re used.
Identifiers must be defined or declared before they’re used.

This statement declares the variables.
Identifiers must be defined or declared before they’re used.

This statement declares the variables.

This is a definition.
MODULE SimpleProgram

EXTENDS Integers
VARIABLES i, pc

\[
Init \triangleq (pc = \text{"start"}) \land (i = 0)
\]

Defines \( Init \) to be equal to

It defines \( Init \) to be equal to
It defines $Init$ to be equal to the initial formula.
It defines \( \text{Init} \) to be equal to the initial formula.

Similarly, this statement
MODULE SimpleProgram

EXTENDS Integers
VARIABLES i, pc

Init $\triangleq (pc = \text{"start"}) \land (i = 0)$

Next $\triangleq \lor \land pc = \text{"start"}$
  $\land i' \in 0..1000$
  $\land pc' = \text{"middle"}$
  $\lor \land pc = \text{"middle"}$
  $\land i' = i + 1$
  $\land pc' = \text{"done"}$

It defines Init to be equal to the initial formula.

Similarly, this statement defines Next to equal
MODULE SimpleProgram

EXTENDS Integers
VARIABLES i, pc

Init $\triangleq (pc = \text{"start"}) \land (i = 0)$

Next $\triangleq \forall \land pc = \text{"start"} \land i' \in 0 \ldots 1000 \land pc' = \text{"middle"}$

$\lor \land pc = \text{"middle"} \land i' = i + 1 \land pc' = \text{"done"}$

Defines Next to equal the next-state formula.

It defines Init to be equal to the initial formula.

Similarly, this statement defines Next to equal the next-state formula.
It defines $\text{Init}$ to be equal to the initial formula.

Similarly, this statement defines $\text{Next}$ to equal the next-state formula.
It defines $Init$ to be equal to the initial formula.

Similarly, this statement defines $Next$ to equal the next-state formula.

You can use any names instead of $Init$ and $Next$,
You can use any names. These are conventional.

It defines $Init$ to be equal to the initial formula.

Similarly, this statement defines $Next$ to equal the next-state formula.

You can use any names instead of $Init$ and $Next$, but they are the ones normally used by convention.
It defines $Init$ to be equal to the initial formula.

Similarly, this statement defines $Next$ to equal the next-state formula.

You can use any names instead of $Init$ and $Next$, but they are the ones normally used by convention.

This is the pretty-printed version of the spec.
--- MODULE SimpleProgram ---

EXTENDS Integers

VARIABLES i, pc

Init == (pc = "start") \(\land\) (i = 0)

Next == \(\lor\) \(\land\) pc = "start"
    \(\land\) i' \(\in\) 0..1000
    \(\land\) pc' = "middle"
\(\lor\) \(\land\) pc = "middle"
    \(\land\) i' = i + 1
    \(\land\) pc' = "done"

---

Here is how you type the spec into the TLA+ Toolbox.

On command, the Toolbox will display
MODULE SimpleProgram

EXTENDS Integers

VARIABLES i, pc

\[ \text{Init} \triangleq (pc = \text{"start"}) \land (i = 0) \]

\[ \text{Next} \triangleq \bigvee \land pc = \text{"start"} \]
\[ \land i' \in 0 \ldots 1000 \]
\[ \land pc' = \text{"middle"} \]
\[ \bigvee \land pc = \text{"middle"} \]
\[ \land i' = i + 1 \]
\[ \land pc' = \text{"done"} \]
DECOMPOSING LARGE SPECS

Decomposing large specs.

[slide 215]
The next-state formula can be 100s of lines.

For real specs, the next-state formula can be hundreds or even thousands of lines.
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We can understand a big formula by splitting it into smaller parts.

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Using definitions.

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Math has a simple and very powerful way to do that: Using definitions.
This spec is too simple to need splitting into parts, but let’s do it anyway.

An obvious way to decompose this spec is
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An obvious way to decompose this spec is by giving names to these two subformulas.

We could call them anything,
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An obvious way to decompose this spec is

by giving names to these two subformulas.

We could call them anything, say Fred and Mary.
But more descriptive names are better, such as

[slide 222]
This spec is too simple to need splitting into parts, but let’s do it anyway.

An obvious way to decompose this spec is by giving names to these two subformulas.

We could call them anything, say Fred and Mary. But more descriptive names are better, such as Pick and Add1.
$\text{Next} \triangleq \forall \wedge pc = \text{“start”}$
$\wedge i' \in 0 \ldots 1000$
$\wedge pc' = \text{“middle”}$
$\forall \wedge pc = \text{“middle”}$
$\wedge i' = i + 1$
$\wedge pc' = \text{“done”}$

So let’s replace this definition of $\text{Next}$
$Pick \triangleq \land pc = \text{"start"} \land i' \in 0 \ldots 1000 \land pc' = \text{"middle"}$

$Add1 \triangleq \land pc = \text{"middle"} \land i' = i + 1 \land pc' = \text{"done"}$

$Next \triangleq Pick \lor Add1$

with these three definitions.
We define \( \text{Pick} \)

\[
\text{Pick} \triangleq \quad \land \ pc = \text{“start”} \\
\land \ i' \in 0 \ldots 1000 \\
\land \ pc' = \text{“middle”}
\]

\[
\text{Add1} \triangleq \quad \land \ pc = \text{“middle”} \\
\land \ i' = i + 1 \\
\land \ pc' = \text{“done”}
\]

\[
\text{Next} \triangleq \text{Pick} \lor \text{Add1}
\]

with these three definitions.

We define \( \text{Pick} \)
with these three definitions.

We define $\textit{Pick}$ and $\textit{Add1}$
We define \( \text{Pick} \) and \( \text{Add1} \) and then define \( \text{Next} \) to equal \( \text{Pick} \) or \( \text{Add1} \) with these three definitions.

\begin{align*}
\text{Pick} & \triangleq \land pc = \text{“start”} \\
& \land i' \in 0 \ldots 1000 \\
& \land pc' = \text{“middle”} \\
\text{Add1} & \triangleq \land pc = \text{“middle”} \\
& \land i' = i + 1 \\
& \land pc' = \text{“done”} \\
\text{Next} & \triangleq \text{Pick} \lor \text{Add1}
\end{align*}
with these three definitions.

We define \( \text{Pick} \) and \( \text{Add1} \) and then define \( \text{Next} \) to equal \( \text{Pick} \) or \( \text{Add1} \). This definition of \( \text{Next} \)
These are equivalent definitions of $Next$.

Is completely equivalent to our original definition.
These are equivalent definitions of $Next$.

Is completely equivalent to our original definition.

It doesn’t matter which one we use.

[slide 231]
This C code example is tiny. Most of the examples I will present are simple.

I believe you’ll learn more by carefully studying simple examples than by skimming complex ones.

For now, you’ll have to trust me — and the engineers at Amazon Web Services and elsewhere who use it — when we say that TLA+ is good for specifying real systems, not just toy examples.
This is the end of Lecture 2 of the TLA+ Video Course

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State Machines in Math