

# TLA<sup>+</sup> Video Course – Lecture 9, Part 2

Leslie Lamport

## THE ALTERNATING BIT PROTOCOL THE PROTOCOL

This video should be viewed in conjunction with a Web page.  
To find that page, search the Web for *TLA<sup>+</sup> Video Course*.

The TLA<sup>+</sup> Video Course  
Lecture 9  
The Alternating Bit Protocol



In this part, we examine the Alternating Bit Protocol itself, and how it implements the liveness property of its high-level specification.

In the process, we learn about strong fairness and some more about using the TLC model checker.

[ slide 2 ]

# **THE SAFETY SPECIFICATION**

## What the Protocol Accomplishes

Remember what the AB protocol is supposed to accomplish.

## What the Protocol Accomplishes

A                      B  
*AVar*:  $\langle \text{"", 1} \rangle$                       *BVar*:  $\langle \text{"", 1} \rangle$

A Sends:

B Receives:

Remember what the AB protocol is supposed to accomplish.

It starts with *AVar* and *BVar* having values like these, where the first component is an arbitrary data item.

## What the Protocol Accomplishes

A                      B  
*AVar*:  $\langle \text{"Fred"}, 0 \rangle$                       *BVar*:  $\langle \text{"", 1} \rangle$

A Sends:        "Fred"

B Receives:

Remember what the AB protocol is supposed to accomplish.

It starts with *AVar* and *BVar* having values like these, where the first component is an arbitrary data item.

**A sends a data item by setting the first element of *AVar* to that item and complementing the one-bit second element.**

## What the Protocol Accomplishes

A

*AVar*:  $\langle \text{"Fred"}, 0 \rangle$

B

*BVar*:  $\langle \text{"Fred"}, 0 \rangle$

A Sends: *"Fred"*

B Receives: *"Fred"*

*B* receives that item.

## What the Protocol Accomplishes

A

*AVar*: *⟨“Mary”, 1⟩*

B

*BVar*: *⟨“Fred”, 0⟩*

A Sends: *“Fred”, “Mary”*

B Receives: *“Fred”*

*B* receives that item.

*A* sends the next data item.

## What the Protocol Accomplishes

A

*AVar*: *⟨“Mary”, 1⟩*

B

*BVar*: *⟨“Mary”, 1⟩*

A Sends: *“Fred”, “Mary”*

B Receives: *“Fred”, “Mary”*

*B* receives that item.

*A* sends the next data item.

And so on.

## What the Protocol Accomplishes

A

*AVar*: *⟨“Mary”, 0⟩*

B

*BVar*: *⟨“Mary”, 1⟩*

A Sends: *“Fred”, “Mary”, “Mary”*

B Receives: *“Fred”, “Mary”*

*B* receives that item.

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And so on.

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*AVar*: *⟨“Mary”, 0⟩*

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B Receives: *“Fred”, “Mary”, “Mary”*

*B* receives that item.

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And so on.

## What the Protocol Accomplishes

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*AVar*: *⟨“Mary”, 0⟩*

B

*BVar*: *⟨“Mary”, 0⟩*

A Sends: *“Fred”, “Mary”, “Mary”, ...*

B Receives: *“Fred”, “Mary”, “Mary”, ...*

*B* receives that item.

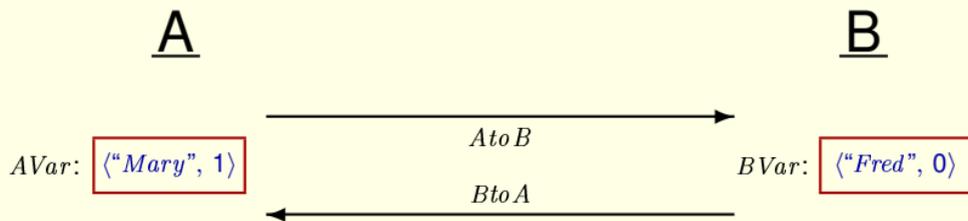
*A* sends the next data item.

And so on.

## How the Protocol Works

Here's how the protocol works.

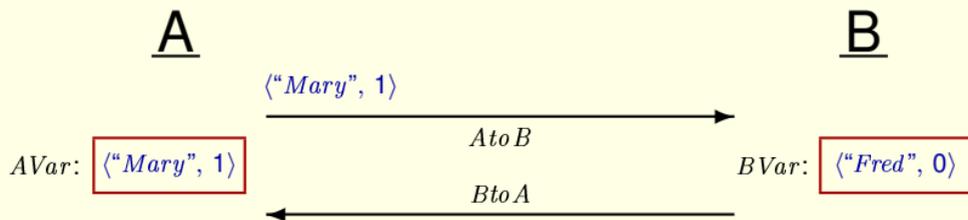
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$A$  and  $B$  communicate over two channels, one from  $A$  to  $B$  and one from  $B$  to  $A$ . The channels can lose messages.

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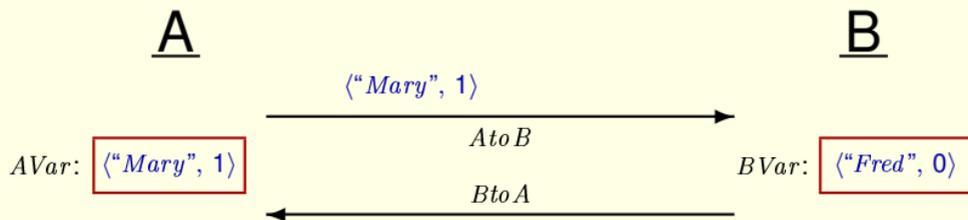


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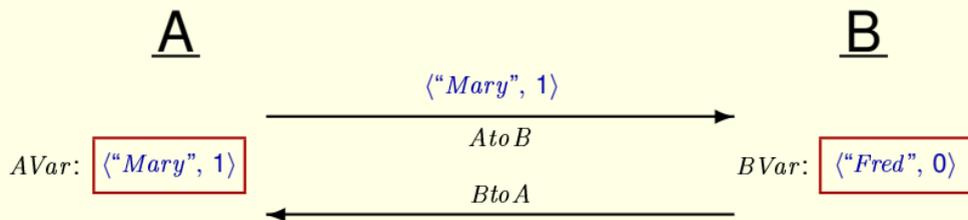


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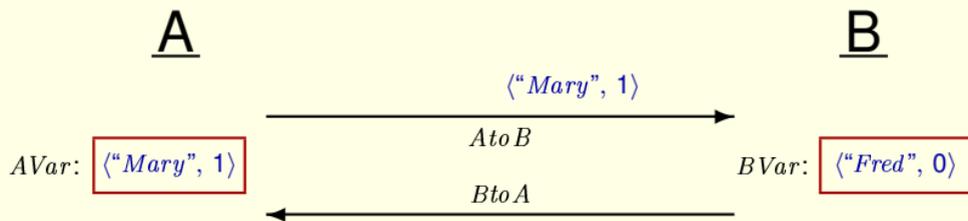


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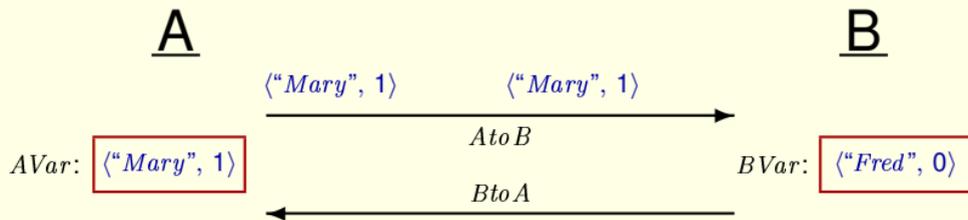


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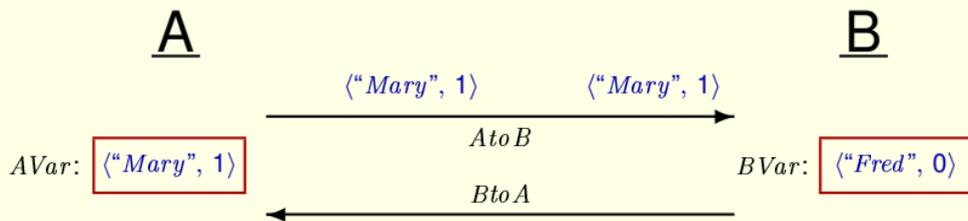
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Since messages can be lost,  $A$  keeps sending its value

## How the Protocol Works



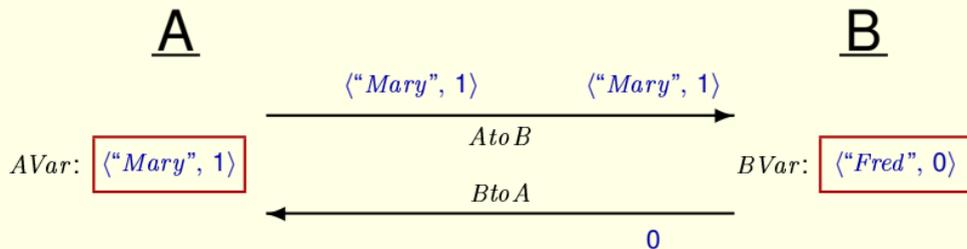
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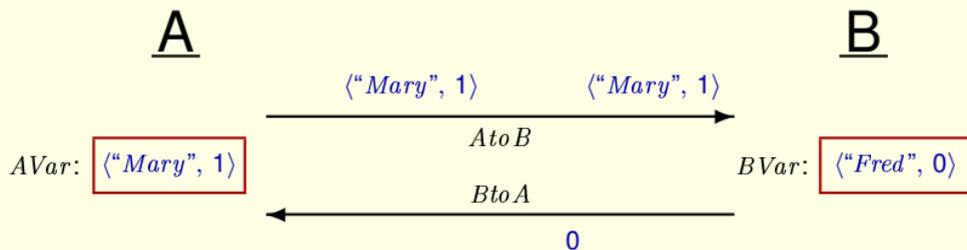
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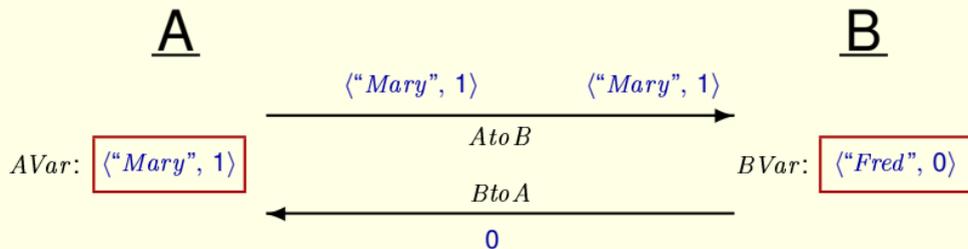
Meanwhile, *B* acknowledges the last value it received by sending its bit.

## How the Protocol Works



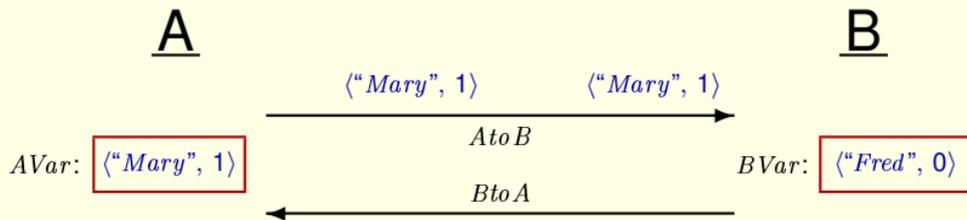
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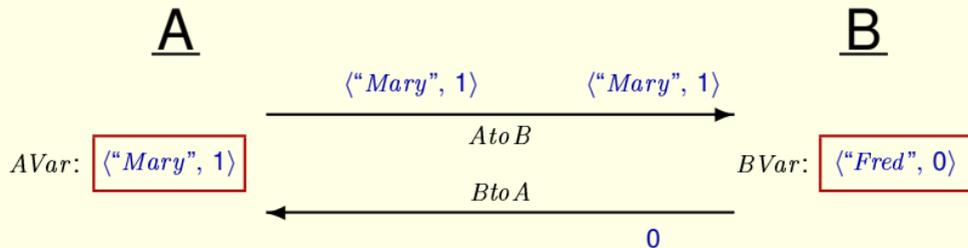
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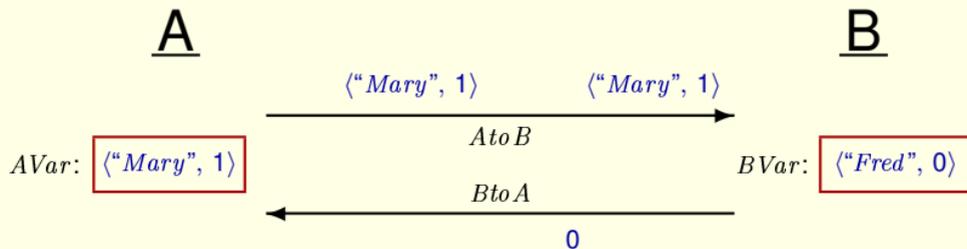
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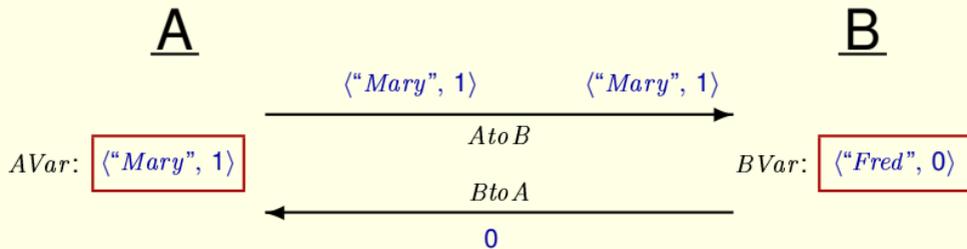
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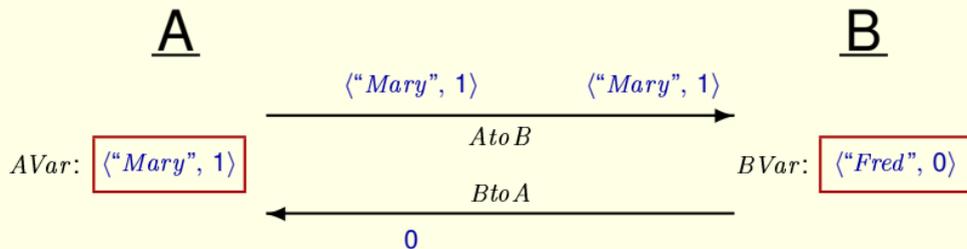
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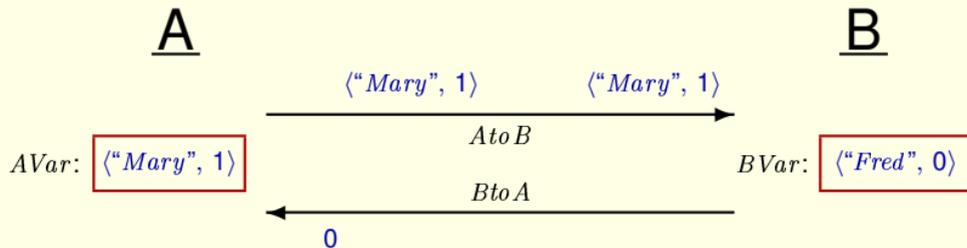
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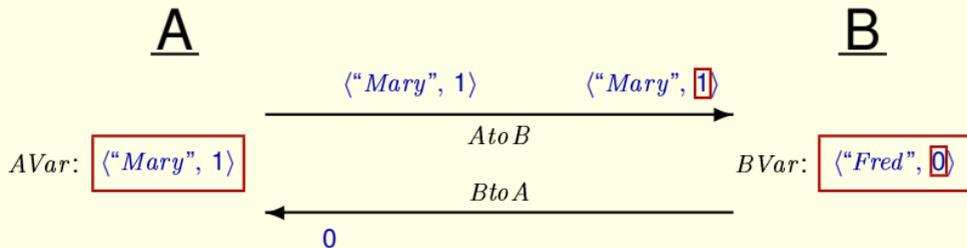
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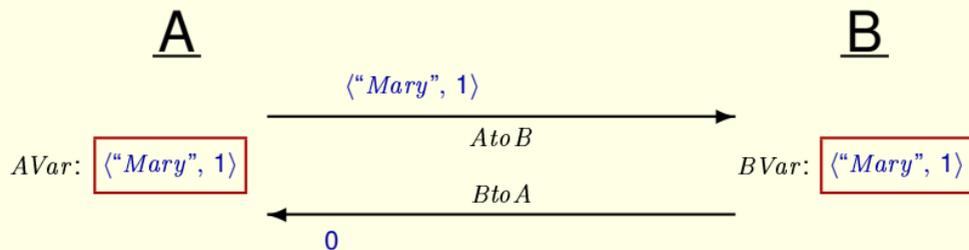


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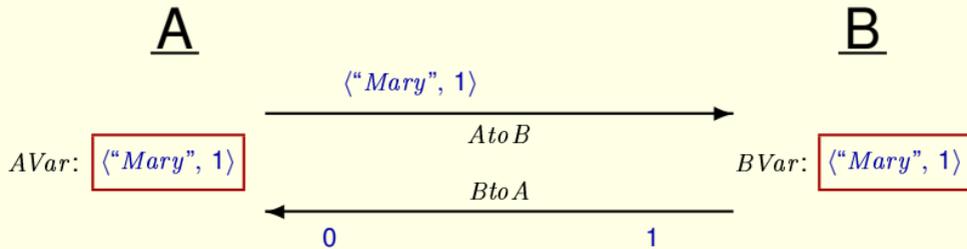
When  $B$  receives the next message on the channel  $A$  to  $B$ , it knows that this is a new value because the message's bit is different from its bit.

## How the Protocol Works



So it changes  $BVar$ .

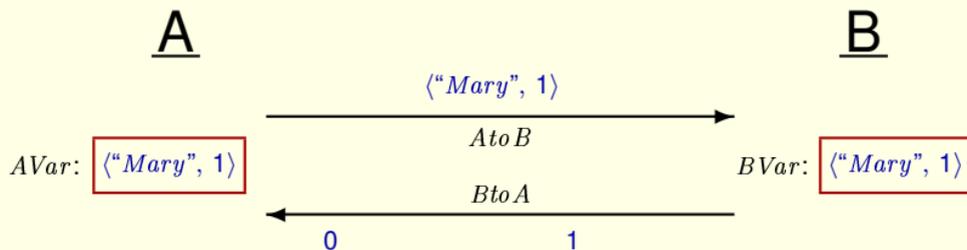
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So it changes  $BVar$ .

It then starts sending its new bit.

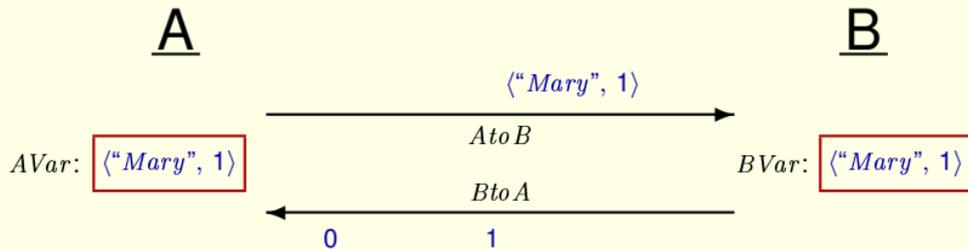
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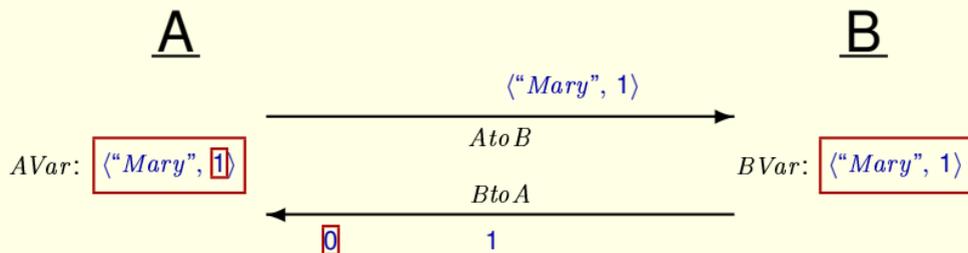
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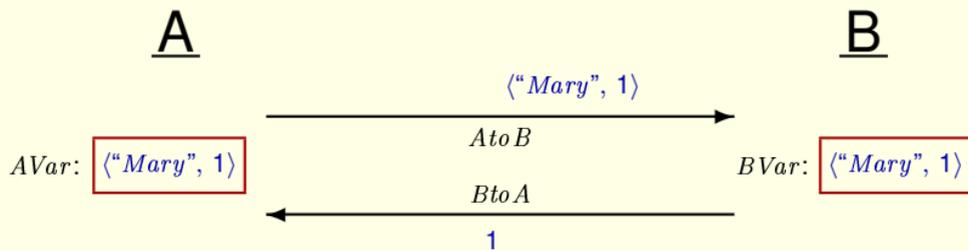


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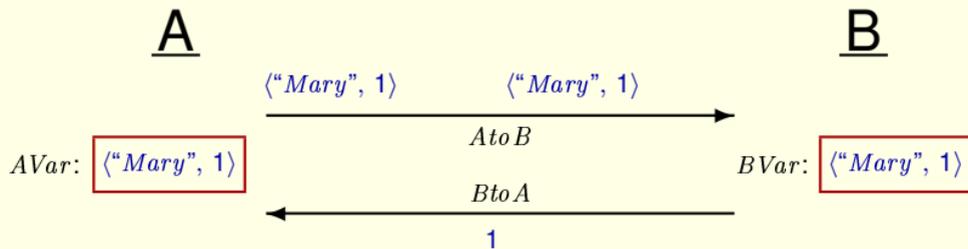
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## How the Protocol Works



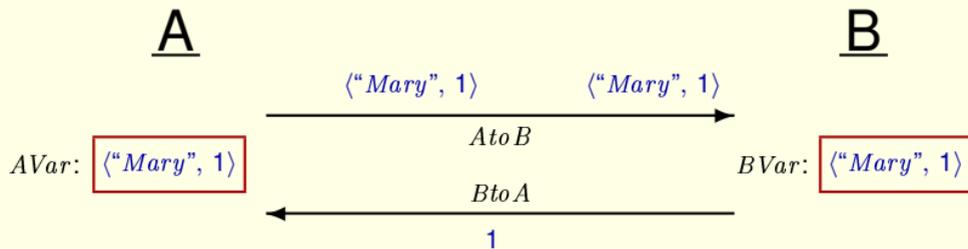
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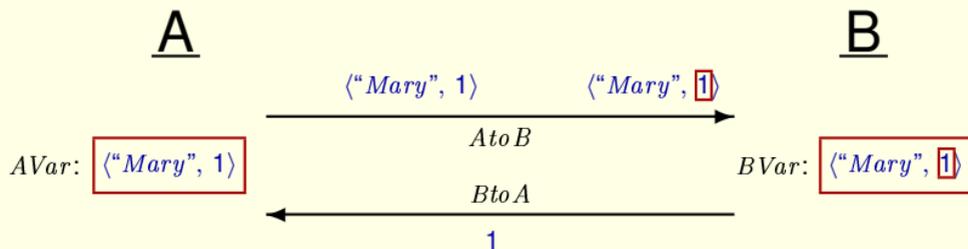
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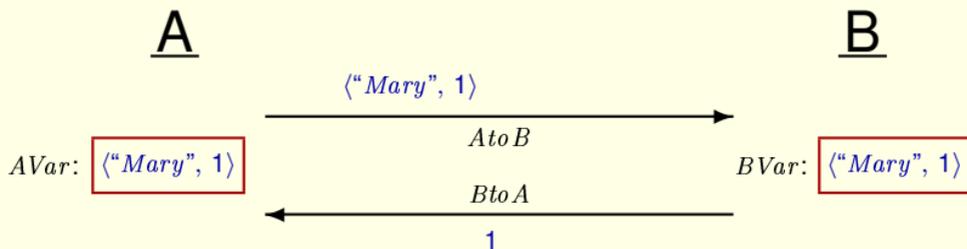
## How the Protocol Works



So  $A$  ignores the message and keeps sending its current value.

Similarly, when  $B$  receives its next message on channel  $A$  to  $B$ , it knows this is a value it has already received because the message's bit is the same as its bit.

## How the Protocol Works

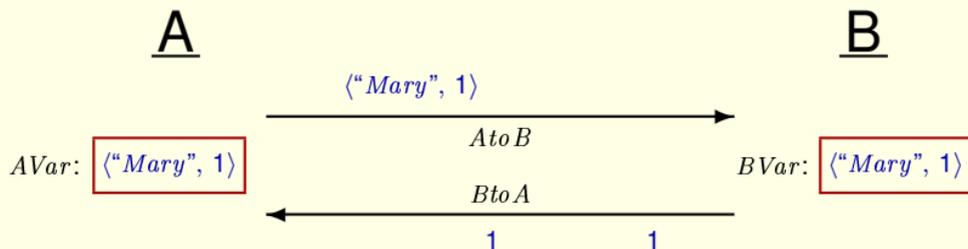


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Similarly, when *B* receives its next message on channel *A* to *B*, it knows this is a value it has already received because the message's bit is the same as its bit.

So *B* ignores the message.

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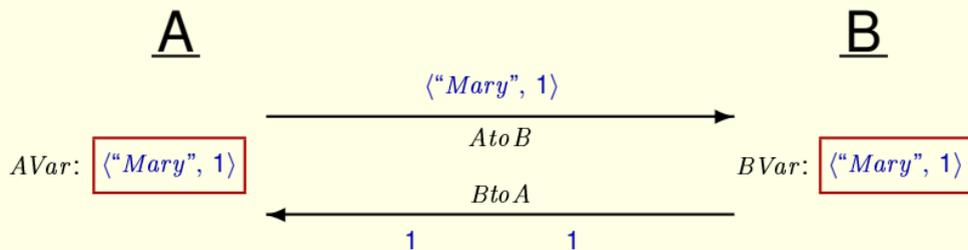


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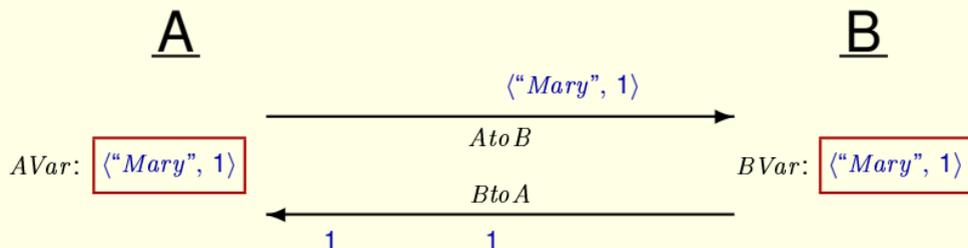


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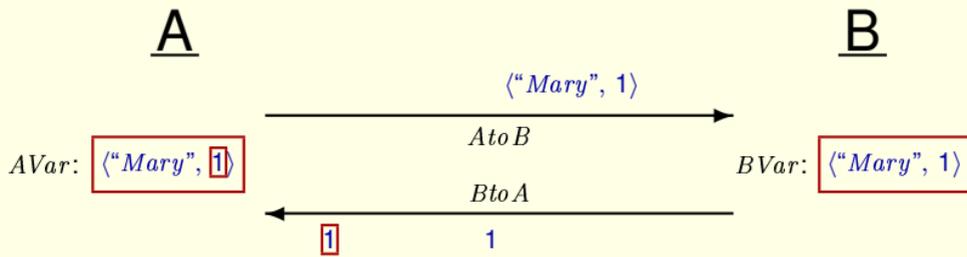


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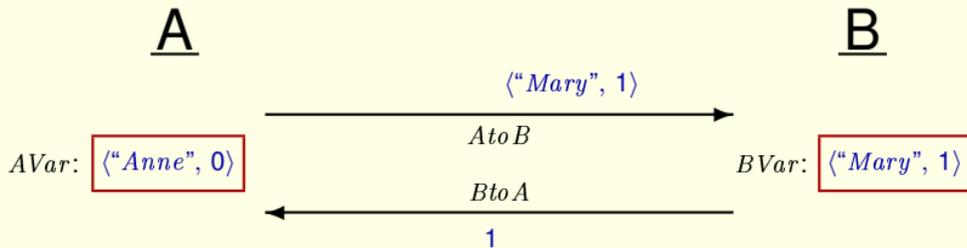
So  $B$  ignores the message and keeps sending its bit.

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When  $A$  receives the next message on the channel  $B$  to  $A$ , it knows that this is an acknowledgement of its current value because the message's bit is the same as its bit.

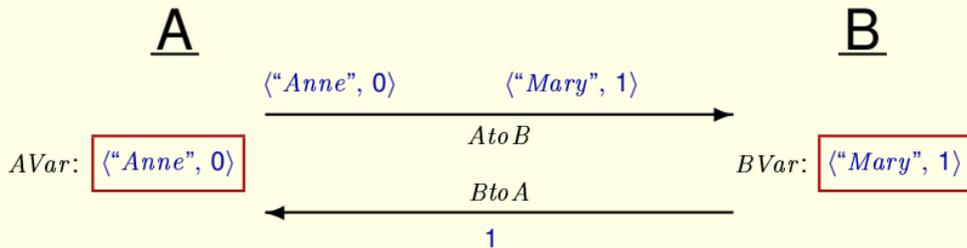
## How the Protocol Works



When  $A$  receives the next message on the channel  $B$  to  $A$ , it knows that this is an acknowledgement of its current value because the message's bit is the same as its bit.

So  $A$  chooses a new data item and flips its bit.

## How the Protocol Works

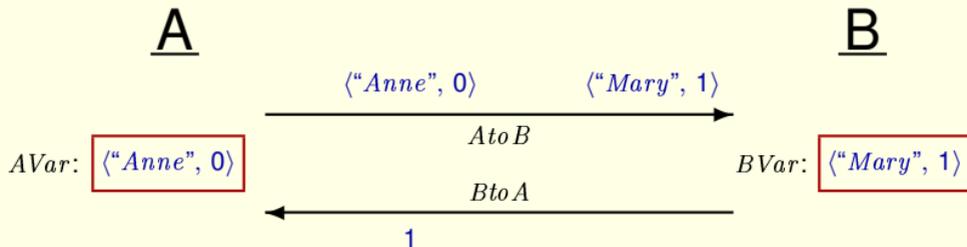


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And so on.

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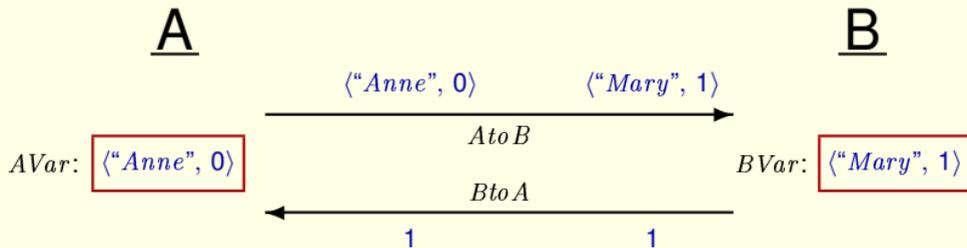


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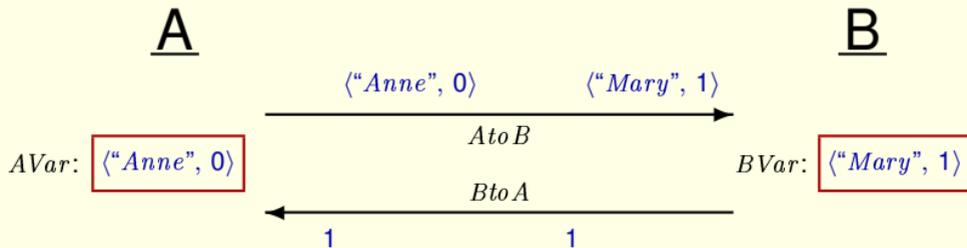


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And so on.

## How the Protocol Works



When *A* receives the next message on the channel *B* to *A*, it knows that this is an acknowledgement of its current value because the message's bit is the same as its bit.

So *A* chooses a new data item and flips its bit.

And so on.

# The TLA<sup>+</sup> Specification

We now look at the safety part of the TLA<sup>+</sup> specification.

## The TLA<sup>+</sup> Specification

Download module  $AB$  and open it in the Toolbox.

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It's in module  $AB$ . Download that spec now and open it in the Toolbox.

## The TLA<sup>+</sup> Specification

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Nothing new except the use of operations on sequences.

We now look at the safety part of the TLA<sup>+</sup> specification.

It's in module  $AB$ . Download that spec now and open it in the Toolbox.

There's nothing new in the safety spec except that it uses the operations on sequences we examined in part one of this lecture.

EXTENDS *Integers, Sequences*

As usual, the module begins with an EXTENDS statement that imports the Integers module

EXTENDS *Integers*, *Sequences*

Imports operators on sequences.

As usual, the module begins with an EXTENDS statement that imports the *Integers* module and the *Sequences* module that defines the operators on sequences.

EXTENDS *Integers, Sequences*

CONSTANT *Data*

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The constant *Data*

EXTENDS *Integers*, *Sequences*

CONSTANT *Data* Same as in *ABSpec*.

As usual, the module begins with an EXTENDS statement that imports the Integers module and the Sequences module that defines the operators on sequences.

The constant *Data* is the same set of data items as in module *ABSpec*.

EXTENDS *Integers, Sequences*

CONSTANT *Data*

*Remove*(*i, seq*)  $\triangleq$

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The constant *Data* is the same set of data items as in module *ABSpec*.

**Remove of *i, seq* was defined in part 1 to equal**

EXTENDS *Integers*, *Sequences*

CONSTANT *Data*

*Remove*(*i*, *seq*)  $\triangleq$  Sequence *seq* with its  
*i*<sup>th</sup> element removed.

As usual, the module begins with an EXTENDS statement that imports the Integers module and the Sequences module that defines the operators on sequences.

The constant *Data* is the same set of data items as in module *ABSpec*.

*Remove* of *i*, *seq* was defined in part 1 to equal sequence *seq* with its *i*<sup>th</sup> element removed.



VARIABLES  $AVar$ ,  $BVar$

$AVar$  and  $BVar$  are the same variables as in  $ABSpec$ ,

VARIABLES  $AVar$ ,  $BVar$ ,  $AtoB$ ,  $BtoA$

$AVar$  and  $BVar$  are the same variables as in  $ABSpec$ , while  $A$  to  $B$  and  $B$  to  $A$  are additional variables that represent the message channels.

VARIABLES  $AVar, BVar, AtoB, BtoA$

$vars \triangleq \langle AVar, BVar, AtoB, BtoA \rangle$

$AVar$  and  $BVar$  are the same variables as in  $ABSpec$ , while  $A$  to  $B$  and  $B$  to  $A$  are additional variables that represent the message channels.

As usual, we define  $vars$  to be the tuple of all variables.

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$TypeOK \triangleq$

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Next is the type-correctness invariant.

VARIABLES  $AVar, BVar, AtoB, BtoA$

$vars \triangleq \langle AVar, BVar, AtoB, BtoA \rangle$

$TypeOK \triangleq \wedge AVar \in Data \times \{0, 1\}$   
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Same as in  $ABSpec$ .

$AVar$  and  $BVar$  are the same variables as in  $ABSpec$ , while  $A$  to  $B$  and  $B$  to  $A$  are additional variables that represent the message channels.

As usual, we define  $vars$  to be the tuple of all variables.

Next is the type-correctness invariant.

The possible values of  $AVar$  and  $BVar$  are the same as in  $ABSpec$ .

VARIABLES  $AVar, BVar, AtoB, BtoA$

$vars \triangleq \langle AVar, BVar, AtoB, BtoA \rangle$

$TypeOK \triangleq \wedge AVar \in Data \times \{0, 1\}$   
 $\wedge BVar \in Data \times \{0, 1\}$   
 $\wedge AtoB \in Seq(Data \times \{0, 1\})$

$AtoB$  is an element of

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The set of sequences of

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The set of sequences of values  $A$  can send.

$AtoB$  is an element of the set of all sequences of values that  $A$  can send.

VARIABLES  $AVar, BVar, AtoB, BtoA$

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$TypeOK \triangleq \wedge AVar \in Data \times \{0, 1\}$

$\wedge BVar \in Data \times \{0, 1\}$

$\wedge AtoB \in Seq(Data \times \{0, 1\})$

**A sends a message by appending it to the end of  $AtoB$ .**

$AtoB$  is an element of the set of all sequences of values that  $A$  can send.

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VARIABLES  $AVar, BVar, AtoB, BtoA$

$vars \triangleq \langle AVar, BVar, AtoB, BtoA \rangle$

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 $\wedge BtoA \in Seq(\{0, 1\})$

The set of sequences of bits

And similarly, the value of  $BtoA$  is always a sequence of bits.

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$Init \triangleq \wedge AVar \in Data \times \{1\}$     Same as in  $ABSpec$   
 $\wedge BVar = AVar$

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 $\wedge AtoB = \langle \rangle$   
 $\wedge BtoA = \langle \rangle$  Channels are empty.

And similarly, the value of  $BtoA$  is always a sequence of bits.

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The subactions of *Next*

The next-state action is the disjunction of five subactions whose definitions come next.

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$$ASnd \triangleq$$

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*A-send* is defined to be

## The subactions of *Next*

$ASnd \triangleq$  **A sends a message.**

The next-state action is the disjunction of five subactions whose definitions come next.

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## The subactions of *Next*

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$ARcv \triangleq$

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*A-receive* is defined to be

## The subactions of *Next*

$ASnd \triangleq$  **A sends a message.**

$ARcv \triangleq$  **A receives a message.**

The next-state action is the disjunction of five subactions whose definitions come next.

*A-send* is defined to be the action of *A* sending a message.

*A-receive* is defined to be the action of *A* receiving a message.

## The subactions of *Next*

$ASnd \triangleq$  A sends a message.

$ARcv \triangleq$  A receives a message.

$BSnd \triangleq$

Similarly for *B-send*

## The subactions of *Next*

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$ARcv \triangleq$  A receives a message.

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Similarly for *B-send*

## The subactions of *Next*

$ASnd \triangleq$  A sends a message.

$ARcv \triangleq$  A receives a message.

$BSnd \triangleq$  B sends a message.

$BRcv \triangleq$

Similarly for *B-send* and *B-receive*.

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Similarly for *B-send* and *B-receive*.

## The subactions of *Next*

$ASnd \triangleq$  A sends a message.

$ARcv \triangleq$  A receives a message.

$BSnd \triangleq$  B sends a message.

$BRcv \triangleq$  B receives a message.

$LoseMsg \triangleq$

Similarly for *B-send* and *B-receive*.

And *Lose-Message* is the action

## The subactions of *Next*

$ASnd \triangleq$  A sends a message.

$ARcv \triangleq$  A receives a message.

$BSnd \triangleq$  B sends a message.

$BRcv \triangleq$  B receives a message.

$LoseMsg \triangleq$  A message is lost.

Similarly for *B-send* and *B-recv*.

And *Lose-Message* is the action that describes losing a message.

$ASnd \triangleq$ 

The definition of *A-send* is simple.

$$ASnd \triangleq \wedge AtoB' = Append(AtoB, AVar)$$

The definition of *A-send* is simple.

It appends the value of *AVar* to the end of the sequence *A-to-B*

$$ASnd \triangleq \wedge AtoB' = Append(AtoB, AVar) \\ \wedge UNCHANGED \langle AVar, BtoA, BVar \rangle$$

The definition of *A-send* is simple.

It appends the value of *AVar* to the end of the sequence *A-to-B*

And leaves all the other variables unchanged.

The action is always enabled.

$$ASnd \triangleq \wedge AtoB' = Append(AtoB, AVar) \\ \wedge UNCHANGED \langle AVar, BtoA, BVar \rangle$$

$$ARcv \triangleq$$

The definition of *A-send* is simple.

It appends the value of *AVar* to the end of the sequence *A-to-B*

And leaves all the other variables unchanged.

The action is always enabled.

The action of *A* receiving a message from *B*

$$ASnd \triangleq \wedge AtoB' = Append(AtoB, AVar) \\ \wedge UNCHANGED \langle AVar, BtoA, BVar \rangle$$

$$ARcv \triangleq \wedge BtoA \neq \langle \rangle$$

is enabled only when the sequence *B-to-A* of messages from *B* is not empty.

$$ASnd \triangleq \wedge AtoB' = Append(AtoB, AVar) \\ \wedge UNCHANGED \langle AVar, BtoA, BVar \rangle$$
$$ARcv \triangleq \wedge BtoA \neq \langle \rangle \\ \wedge IF \text{Head}(BtoA) = AVar[2]$$

THEN

ELSE

is enabled only when the sequence *B-to-A* of messages from *B* is not empty.

If the bit at the head of *B-to-A* equals *AVar*'s bit, so *B* is acknowledging *AVar*'s current value,

$$ASnd \triangleq \wedge AtoB' = Append(AtoB, AVar) \\ \wedge UNCHANGED \langle AVar, BtoA, BVar \rangle$$
$$ARcv \triangleq \wedge BtoA \neq \langle \rangle \\ \wedge \text{IF } Head(BtoA) = AVar[2] \\ \text{THEN } \exists d \in Data : \\ \quad \boxed{AVar'} = \langle d, 1 - AVar[2] \rangle \\ \text{ELSE}$$

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If the bit at the head of *B-to-A* equals *AVar*'s bit, so *B* is acknowledging *AVar*'s current value,  
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$$ASnd \triangleq \wedge AtoB' = Append(AtoB, AVar) \\ \wedge UNCHANGED \langle AVar, BtoA, BVar \rangle$$
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is enabled only when the sequence *B-to-A* of messages from *B* is not empty.

If the bit at the head of *B-to-A* equals *AVar*'s bit, so *B* is acknowledging *AVar*'s current value,  
then the new value of *AVar* is set just like in the *A* action of *ABSpec*: to a pair whose first element is a non-deterministically chosen element of *Data*,

$$ASnd \triangleq \wedge AtoB' = Append(AtoB, AVar) \\ \wedge UNCHANGED \langle AVar, BtoA, BVar \rangle$$
$$ARcv \triangleq \wedge BtoA \neq \langle \rangle \\ \wedge \text{IF } Head(BtoA) = AVar[2] \\ \text{THEN } \exists d \in Data : \\ \quad AVar' = \langle d, 1 - AVar[2] \rangle \\ \text{ELSE}$$

and whose second element is the complement of the current value of  $AVar$ 's bit.

$$ASnd \triangleq \wedge AtoB' = Append(AtoB, AVar) \\ \wedge UNCHANGED \langle AVar, BtoA, BVar \rangle$$
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and whose second element is the complement of the current value of  $AVar$ 's bit.

Otherwise,  $AVar$  is unchanged.

$$ASnd \triangleq \wedge AtoB' = Append(AtoB, AVar) \\ \wedge UNCHANGED \langle AVar, BtoA, BVar \rangle$$

$$ARcv \triangleq \wedge BtoA \neq \langle \rangle \\ \wedge \text{IF } Head(BtoA) = AVar[2] \\ \text{THEN } \exists d \in Data : \\ \quad AVar' = \langle d, 1 - AVar[2] \rangle \\ \text{ELSE } AVar' = AVar \\ \wedge BtoA' = Tail(BtoA)$$

and whose second element is the complement of the current value of  $AVar$ 's bit.

Otherwise,  $AVar$  is unchanged.

And the message  $A$  is receiving, which is at the head of the sequence  $B$ -to- $A$ , is removed from  $B$ -to- $A$ .

$$BSnd \triangleq \wedge BtoA' = Append(BtoA, BVar[2]) \\ \wedge \text{UNCHANGED } \langle AVar, BVar, AtoB \rangle$$

$$BRcv \triangleq \wedge AtoB \neq \langle \rangle \\ \wedge \text{IF } Head(AtoB)[2] \neq BVar[2] \\ \quad \text{THEN } BVar' = Head(AtoB) \\ \quad \text{ELSE } BVar' = BVar \\ \wedge AtoB' = Tail(AtoB) \\ \wedge \text{UNCHANGED } \langle AVar, BtoA \rangle$$

The definitions of *BSnd* and *BRcv* are similar; you can read them yourself.

*LoseMsg*  $\triangleq$

Next comes the definition of *Lose Message*.

$LoseMsg \triangleq \wedge \vee$  Remove a message from  $AtoB$ .

$\vee$  Remove a message from  $BtoA$ .

$\wedge$  UNCHANGED  $\langle AVar, BVar \rangle$

Next comes the definition of *Lose Message*.

It removes a message from  $AtoB$  or  $BtoA$  and leaves  $AVar$  and  $BVar$  unchanged.

$$\text{LoseMsg} \triangleq \wedge \vee \wedge \exists i \in 1 \dots \text{Len}(AtoB) :$$
$$\vee \text{ Remove a message from } BtoA .$$
$$\wedge \text{ UNCHANGED } \langle AVar, BVar \rangle$$

Next comes the definition of *Lose Message*.

It removes a message from *AtoB* or *BtoA* and leaves *AVar* and *BVar* unchanged.

The formula that describes removing a message from *AtoB* asserts that for some *i* between 1 and the length of the sequence *AtoB*

$$\text{LoseMsg} \triangleq \bigwedge \bigvee \bigwedge \exists i \in 1 \dots \text{Len}(\text{AtoB}) : \\ \text{AtoB}' = \text{Remove}(i, \text{AtoB})$$

$\bigvee$  Remove a message from *BtoA*.

$$\bigwedge \text{UNCHANGED} \langle \text{AVar}, \text{BVar} \rangle$$

the new value of *AtoB* is the sequence obtained by removing the  $i^{\text{th}}$  element from the current value of *AtoB*.

$$\begin{aligned}
 LoseMsg \triangleq & \quad \wedge \vee \wedge \exists i \in 1 .. Len(Atob) : \\
 & \quad \quad \quad Atob' = Remove(i, Atob) \\
 & \quad \quad \quad \wedge BtoA' = BtoA \\
 & \quad \vee \text{ Remove a message from } BtoA.
 \end{aligned}$$

$$\wedge \text{ UNCHANGED } \langle AVar, BVar \rangle$$

the new value of *Atob* is the sequence obtained by removing the *i*<sup>th</sup> element from the current value of *Atob*.

And *BtoA* is unchanged.

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 LoseMsg \triangleq & \quad \wedge \vee \wedge \exists i \in 1 .. Len(Atob) : \\
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 & \quad \quad \wedge BtoA' = BtoA \\
 & \quad \vee \text{ Remove a message from } BtoA.
 \end{aligned}$$

$$\wedge \text{ UNCHANGED } \langle AVar, BVar \rangle$$

the new value of *Atob* is the sequence obtained by removing the  $i^{\text{th}}$  element from the current value of *Atob*.

And *BtoA* is unchanged.

The formula that describes removing a message from *BtoA*

$$\begin{aligned}
LoseMsg \triangleq & \quad \wedge \vee \wedge \exists i \in 1 .. Len(AtoB) : \\
& \quad \quad \quad AtoB' = Remove(i, AtoB) \\
& \quad \quad \quad \wedge BtoA' = BtoA \\
& \quad \vee \wedge \exists i \in 1 .. Len(BtoA) : \\
& \quad \quad \quad BtoA' = Remove(i, BtoA) \\
& \quad \quad \quad \wedge AtoB' = AtoB \\
& \quad \wedge UNCHANGED \langle AVar, BVar \rangle
\end{aligned}$$

the new value of *AtoB* is the sequence obtained by removing the *i*<sup>th</sup> element from the current value of *AtoB*.

And *BtoA* is unchanged.

The formula that describes removing a message from *BtoA* is similar.

$$Next \triangleq ASnd \vee ARcv \vee BSnd \vee BRcv \vee LoseMsg$$

Then comes the definition of *Next*

$$Next \triangleq ASnd \vee ARcv \vee BSnd \vee BRcv \vee LoseMsg$$

$$Spec \triangleq Init \wedge \square[Next]_{vars}$$

Then comes the definition of *Next*  
and the standard safety specification.

# CHECKING SAFETY

☰ What is the behavior spec?

Initial predicate and next-state relation

Init:

Next:

Temporal formula

No Behavior Spec

Create a new model with the default specification  $Spec$ ,

What is the behavior spec?

Initial predicate and next-state relation

Init:

Next:

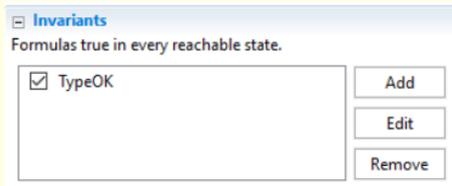
Temporal formula

No Behavior Spec

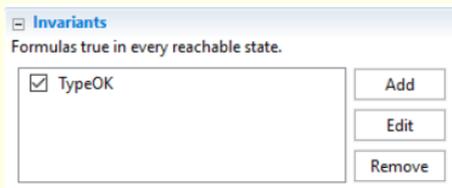
What is the model?

Specify the values of declared constants.

Create a new model with the default specification *Spec*,  
letting *Data* be a small set of model values.



Have TLC check that *TypeOK* is an invariant.



But don't run TLC yet.

Have TLC check that *TypeOK* is an invariant.

But don't run it yet.

A and B can keep sending messages faster than they get lost or received.

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There is no limit to how long the sequences  $AtoB$  and  $BtoA$  can be.

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**There are infinitely many reachable states**

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The specification allows infinitely many reachable states, and since TLC tries to compute all reachable states,

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There is no limit to how long the sequences  $AtoB$  and  $BtoA$  can be.

There are infinitely many reachable states, so TLC will run forever.

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**We could change the spec to limit the lengths of  $AtoB$  and  $BtoA$**

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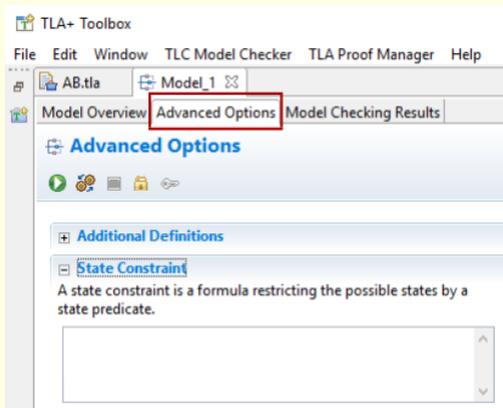
There are infinitely many reachable states, so TLC will run forever.

**We could change the spec to limit the lengths of  $AtoB$  and  $BtoA$ , but we shouldn't have to change the specification to model check it.**

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We can tell TLC to examine only states  
where  $AtoB$  and  $BtoA$  are not too long.

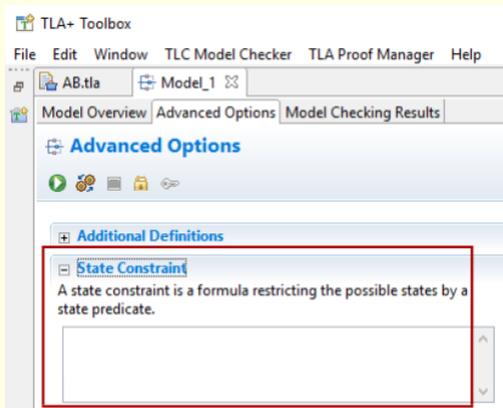
Here's how we can tell TLC to examine only states in which  $AtoB$  and  $BtoA$   
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On the model's advanced options page,

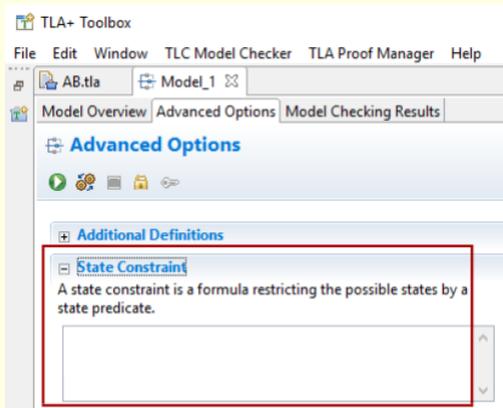
[slide 118]



Here's how we can tell TLC to examine only states in which  $AtoB$  and  $BtoA$  aren't too long.

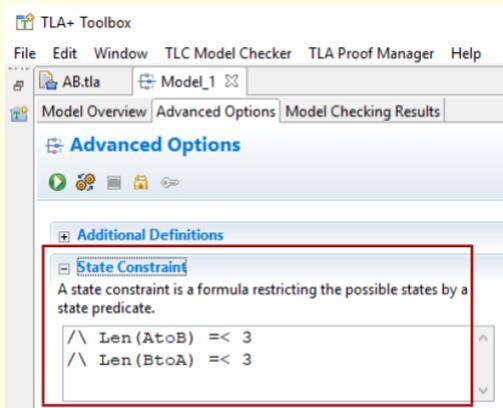
On the model's advanced options page, go to the *state constraint* section.

[slide 119]



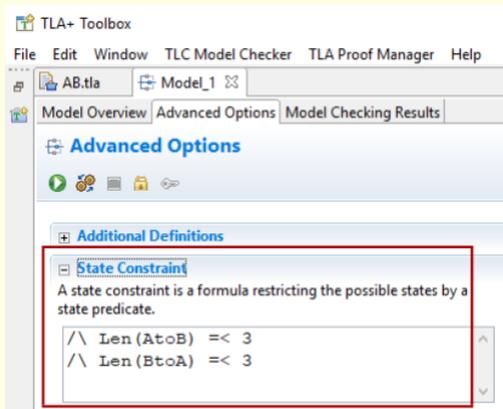
Tell TLC to examine only states with  
 $Len(AtoB)$  and  $Len(BtoA)$  at most 3.

For example, you can tell TLC to examine only states in which the lengths of  
 $AtoB$  and  $BtoA$  are at most 3,



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For example, you can tell TLC to examine only states in which the lengths of  $AtoB$  and  $BtoA$  are at most 3, by entering this state formula.

To understand exactly what this does

## How TLC Computes Reachable States

you need to understand how TLC computes reachable states when it has no state constraint.

## How TLC Computes Reachable States



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**Starting from the set of initial states.**

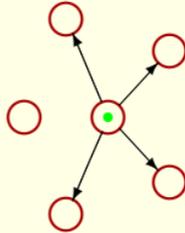
## How TLC Computes Reachable States



you need to understand how TLC computes reachable states when it has no state constraint.

Starting from the set of initial states. **It chooses one.**

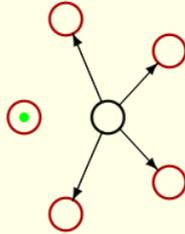
## How TLC Computes Reachable States



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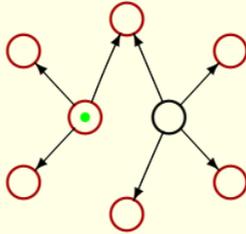
Starting from the set of initial states. It chooses one. and computes all possible next states from that state.

## How TLC Computes Reachable States



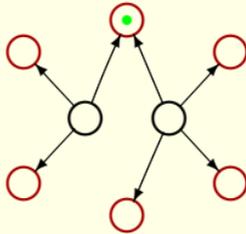
It then chooses another state to explore.

## How TLC Computes Reachable States



It then chooses another state to explore. and finds all possible next states from it.

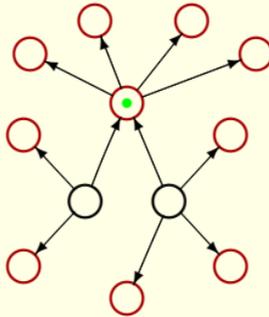
## How TLC Computes Reachable States



It then chooses another state to explore. and finds all possible next states from it.

It then chooses another unexplored state

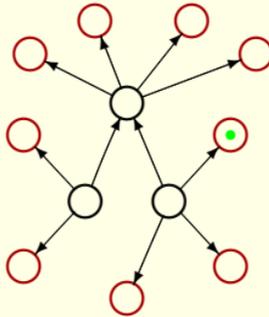
## How TLC Computes Reachable States



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## How TLC Computes Reachable States

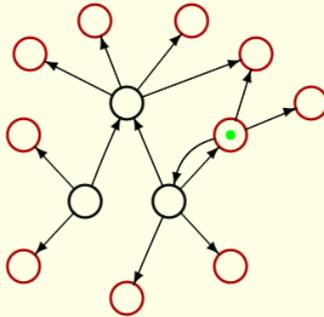


It then chooses another state to explore. and finds all possible next states from it.

It then chooses another unexplored state and finds its next states.

And it keeps on doing this.

## How TLC Computes Reachable States

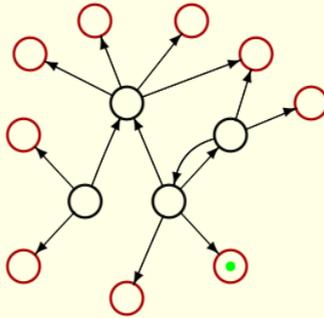


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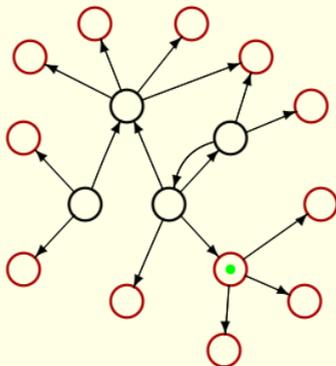


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## How TLC Computes Reachable States

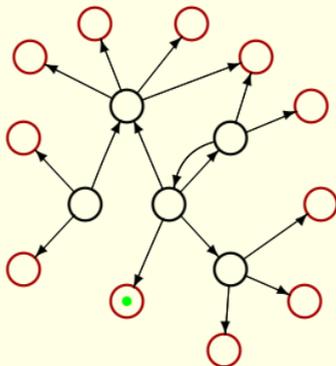


It then chooses another state to explore. and finds all possible next states from it.

It then chooses another unexplored state and finds its next states.

And it keeps on doing this.

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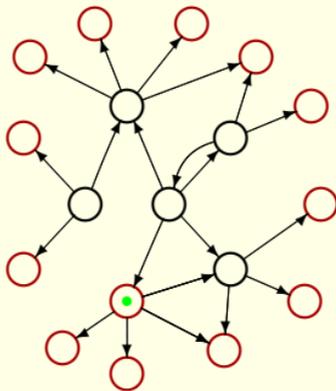


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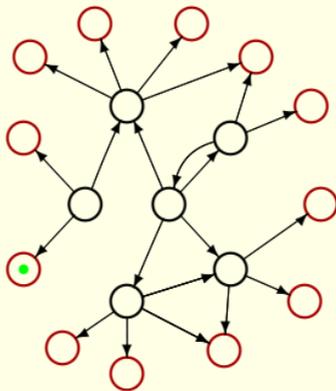


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And it keeps on doing this.

And so on, until it has explored all reachable states.

## How TLC Uses a Constraint

Now here's how TLC computes reachable states when it *has* a state constraint.

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Starting from the set of initial states.

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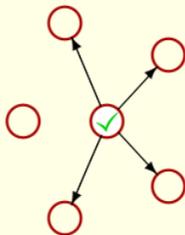


Now here's how TLC computes reachable states when it *has* a state constraint.

Starting from the set of initial states. It chooses one and then checks if the state satisfies the constraint.

Let's suppose it does.

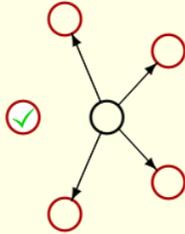
## How TLC Uses a Constraint



As before, TLC then computes all possible next states from that state

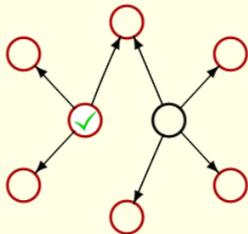


## How TLC Uses a Constraint



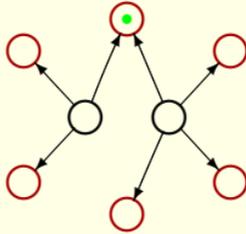
As before, TLC then computes all possible next states from that state and chooses another state to explore. It checks if *that* state satisfies the constraint. Again, let's suppose it does.

## How TLC Uses a Constraint



TLC then finds all possible next states from it.

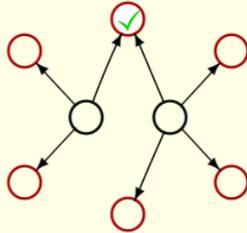
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It keeps going like this

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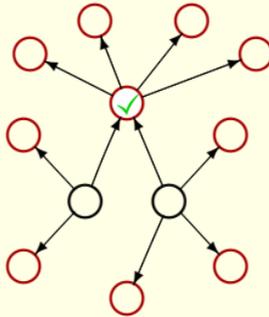


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It keeps going like this

As long as it finds states that satisfy the constraint.

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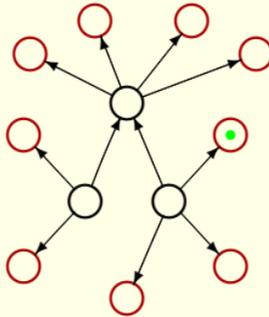


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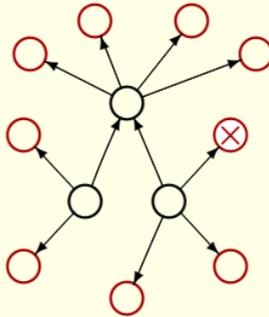


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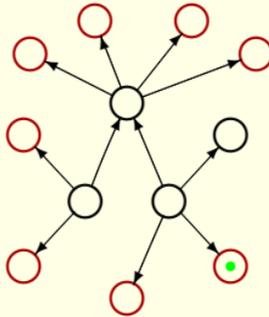
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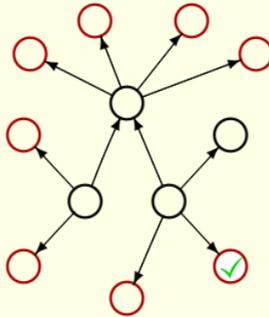
**Suppose it now finds a state that doesn't satisfy the constraint.**

## How TLC Uses a Constraint



It doesn't explore further from that state and instead just goes on to the next unexplored state,

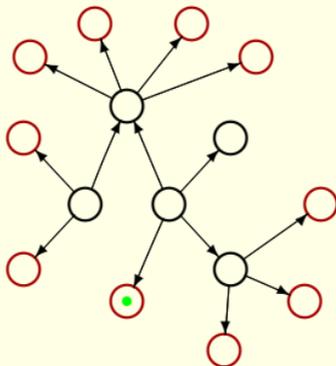
## How TLC Uses a Constraint



It doesn't explore further from that state and instead just goes on to the next unexplored state, exploring that state if it satisfies the constraint.

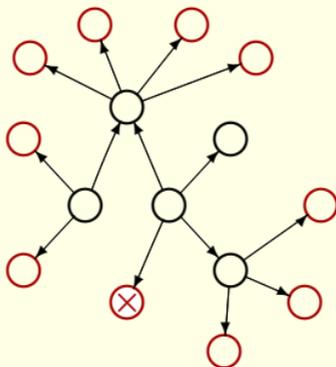


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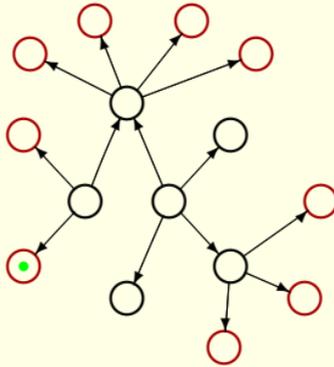
## How TLC Uses a Constraint



It doesn't explore further from that state and instead just goes on to the next unexplored state, exploring that state if it satisfies the constraint.

And continuing like that, exploring only states that satisfy the constraint,

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It doesn't explore further from that state and instead just goes on to the next unexplored state, exploring that state if it satisfies the constraint.

And continuing like that, exploring only states that satisfy the constraint, **until** it finds no more states to explore.

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The AB protocol should implement its high-level specification, so formula  $Spec$  of module  $AB$  should imply formula  $Spec$  of module  $ABS_{spec}$ .

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INSTANCE  $ABS_{spec}$

is illegal in module  $AB$  because it imports definitions of  $Spec, \dots$ , which are already defined in  $AB$ .

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$ABS \triangleq \text{INSTANCE } ABSpec$

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Imports definitions of  $Spec, \dots$  from  $ABSpec$   
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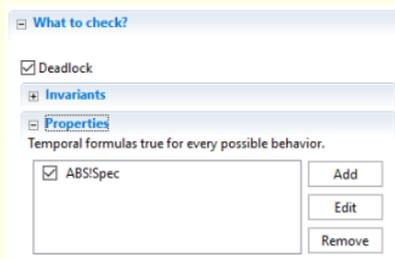
**THEOREM**  $Spec \Rightarrow ABS!Spec$

This theorem states that the safety specification of the alternating bit protocol implements its high-level safety specification from module  $ABSpec$ .

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This theorem states that the safety specification of the alternating bit protocol implements its high-level safety specification from module  $ABSpec$ .

TLC will verify it by checking that specification  $Spec$  satisfies the temporal property A-B-S bang spec .

# LIVENESS

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Which means that this theorem should be true.

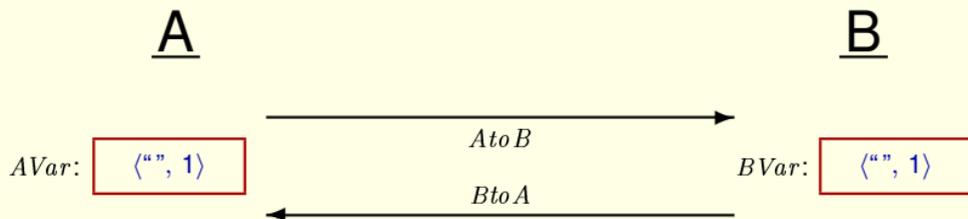
$$\mathit{FairSpec} \triangleq \mathit{Spec} \wedge \text{fairness properties}$$

Which means that this theorem should be true.

$$\text{FairSpec} \stackrel{\Delta}{=} \text{Spec} \wedge \text{WF}_{\text{vars}}(\text{Next})$$

Weak fairness of the *Next* action doesn't work.

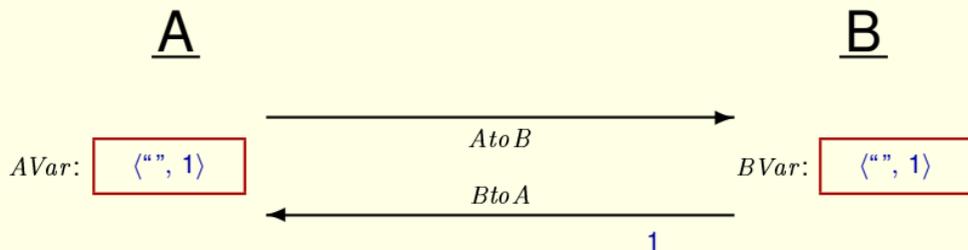
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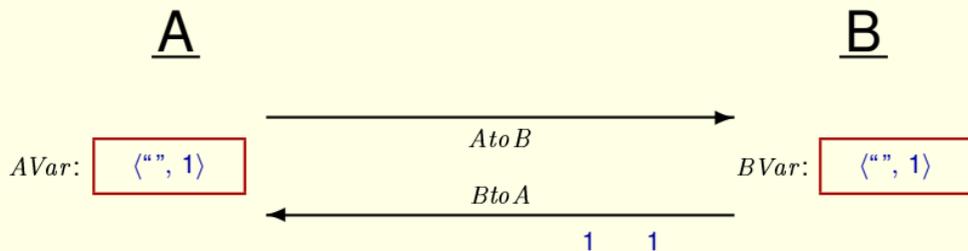
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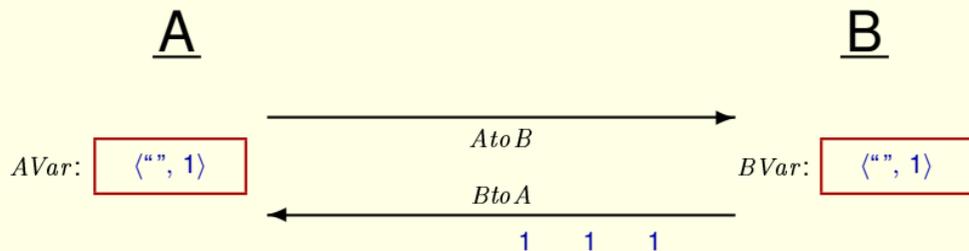
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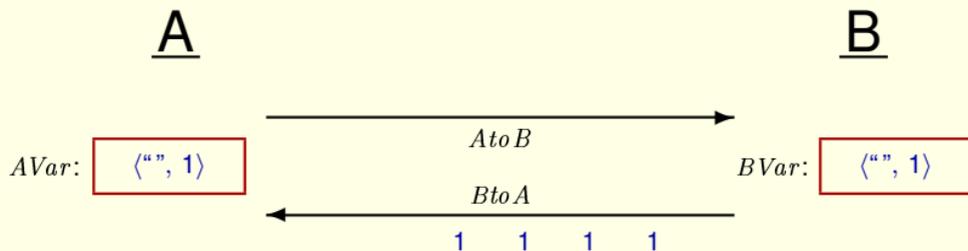
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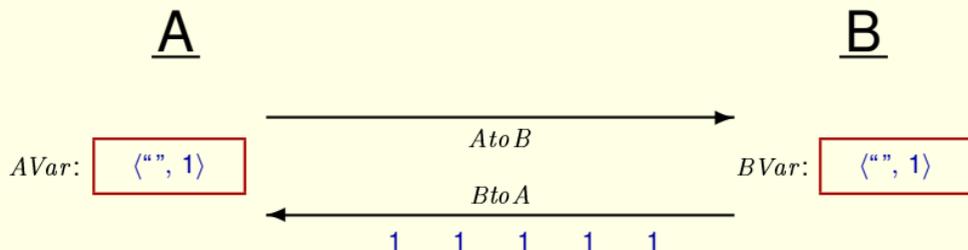


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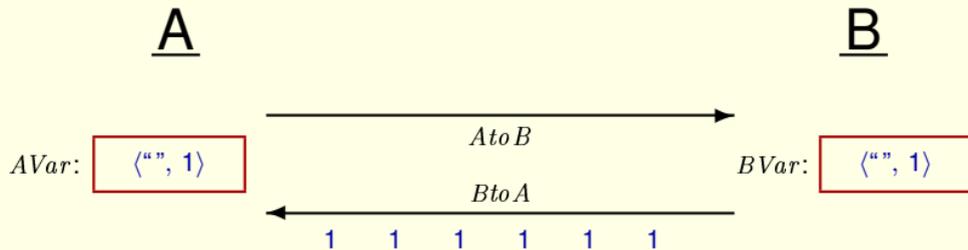


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Weak fairness of the *Next* action doesn't work.

For example, it allows a behavior in which B just keeps sending acknowledgments

and nothing else ever happens.

So we need a stronger fairness property.

$$FairSpec \triangleq Spec \wedge \text{fairness properties}$$
$$Next \triangleq ASnd \vee ARcv \vee BSnd \vee BRcv \vee LoseMsg$$

Remember the definition of the next-state action.

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We need separate fairness requirements on these four subactions, to make sure that each of them keeps being executed.

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We don't want any fairness requirement on the Lose-Message action because we don't want to require that messages have to be lost.

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So, let's try weak fairness of these actions.

$$\text{FairSpec} \triangleq \text{Spec} \wedge \mathbf{SF}_{\text{vars}}(\text{ARcv}) \wedge \mathbf{SF}_{\text{vars}}(\text{BRcv}) \wedge \\ \mathbf{WF}_{\text{vars}}(\text{ASnd}) \wedge \mathbf{WF}_{\text{vars}}(\text{BSnd})$$

Module  $AB$  contains this definition.

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THEOREM  $\text{FairSpec} \Rightarrow \text{ABS!FairSpec}$

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This is a plausible specification, so let's check if it satisfies this theorem.

Clone your model (removing any symmetry set).

Make a clone of the model you used before (removing any symmetry set).

Clone your model (removing any symmetry set).

Modify the specification and property to check.

The screenshot shows a configuration window for a model checker. At the top, there is a radio button labeled "Temporal formula" which is selected. Below it is a text input field containing "FairSpec". To the right of the input field are up and down arrow icons. Below this is another radio button labeled "No Behavior Spec" which is unselected. A section titled "What to check?" is expanded, showing a list of checkboxes. The "Deadlock" checkbox is checked. Below it, the "Invariants" section is collapsed. The "Properties" section is expanded, showing a text input field containing "ABS!FairSpec" with a checked checkbox to its left. To the right of the input field are three buttons: "Add", "Edit", and "Remove". Below the input field is the text "Temporal formulas true for every possible behavior."

Make a clone of the model you used before (removing any symmetry set).

In the clone, modify the specification and property to check by replacing *Spec* with *FairSpec*.

Run TLC on the model.

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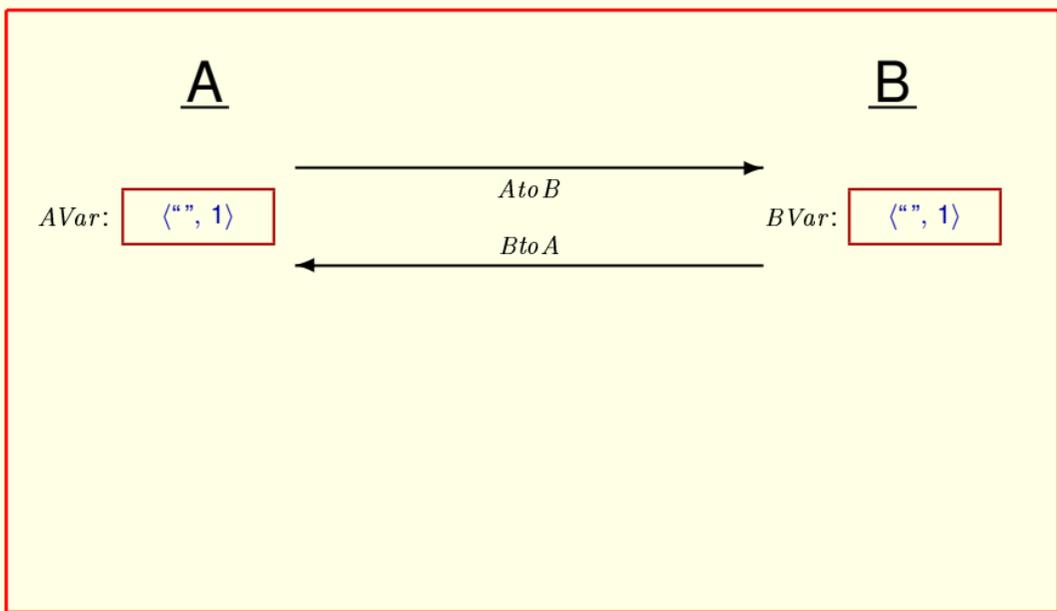
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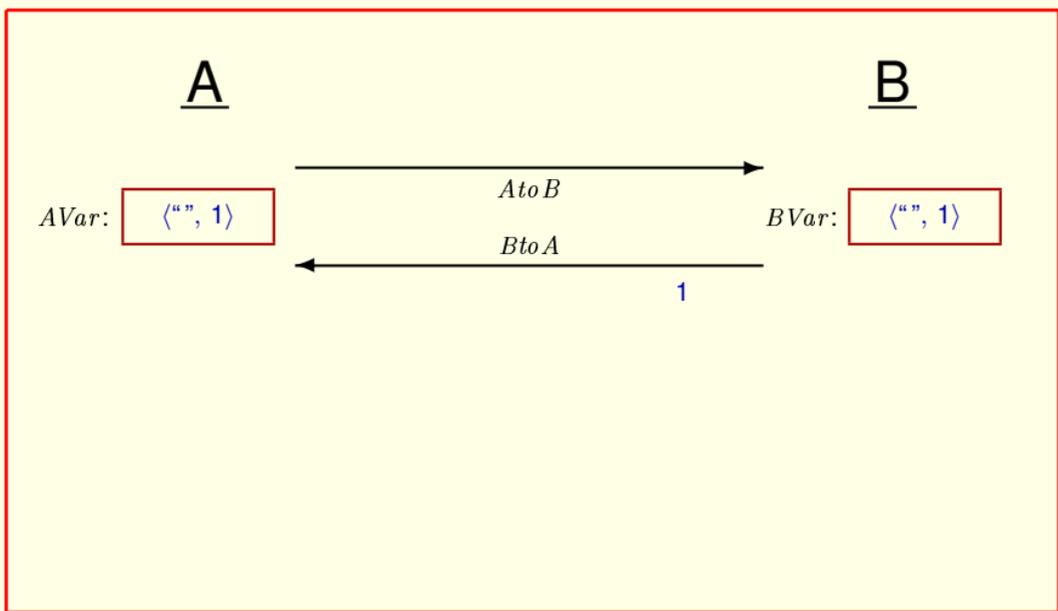
It reports that the temporal property was violated and produces a counterexample.

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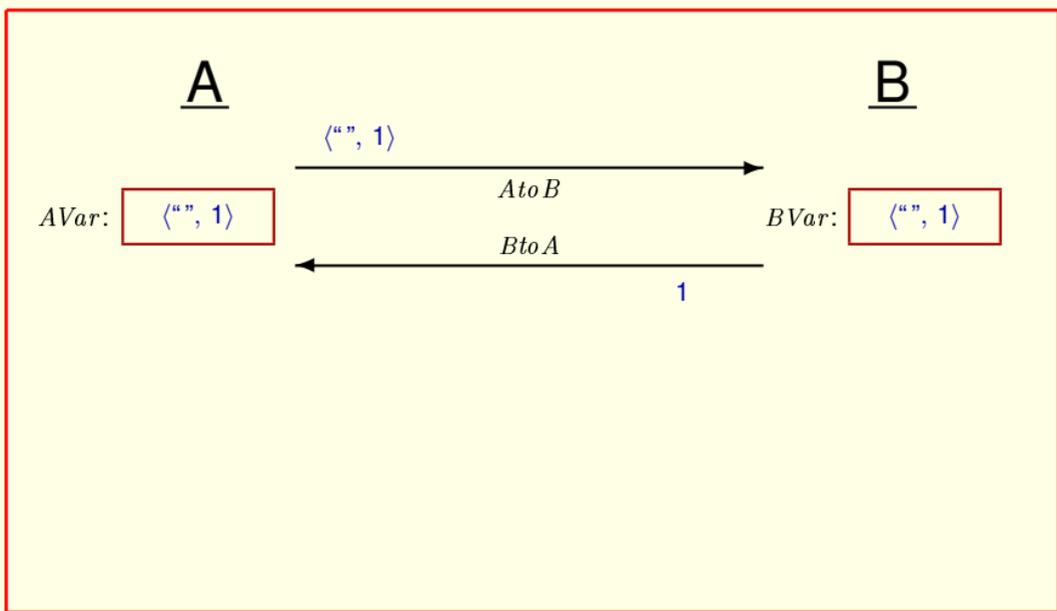


Here's the counterexample that TLC finds.



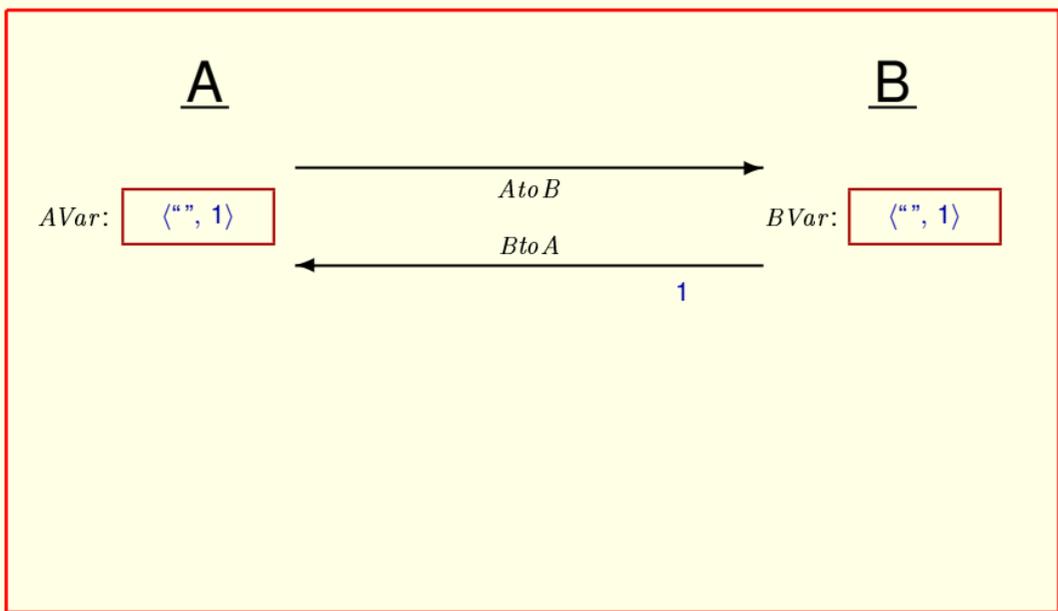
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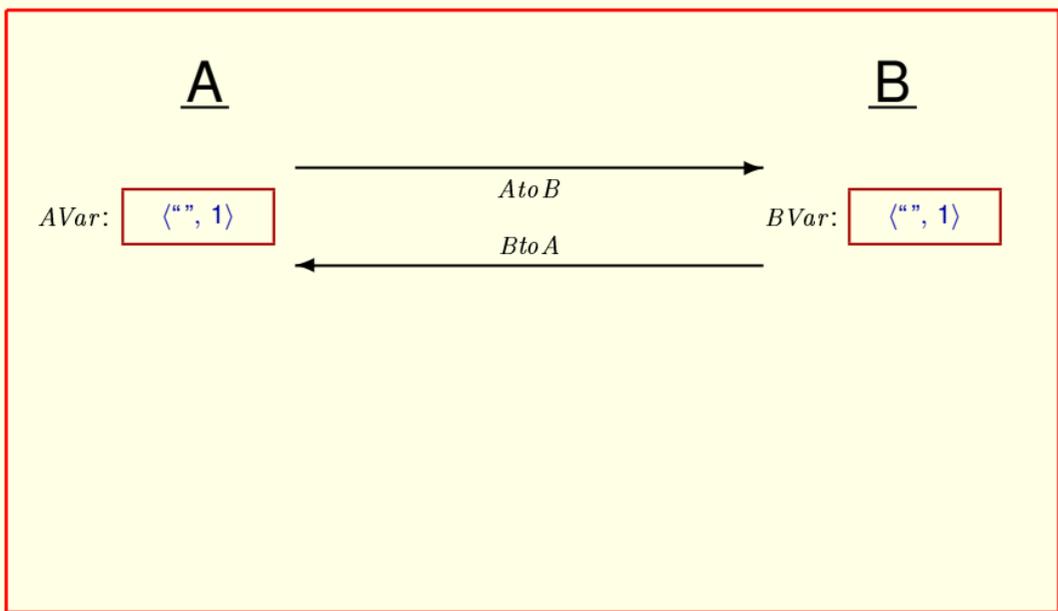
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B sends an acknowledgment, A sends its value,



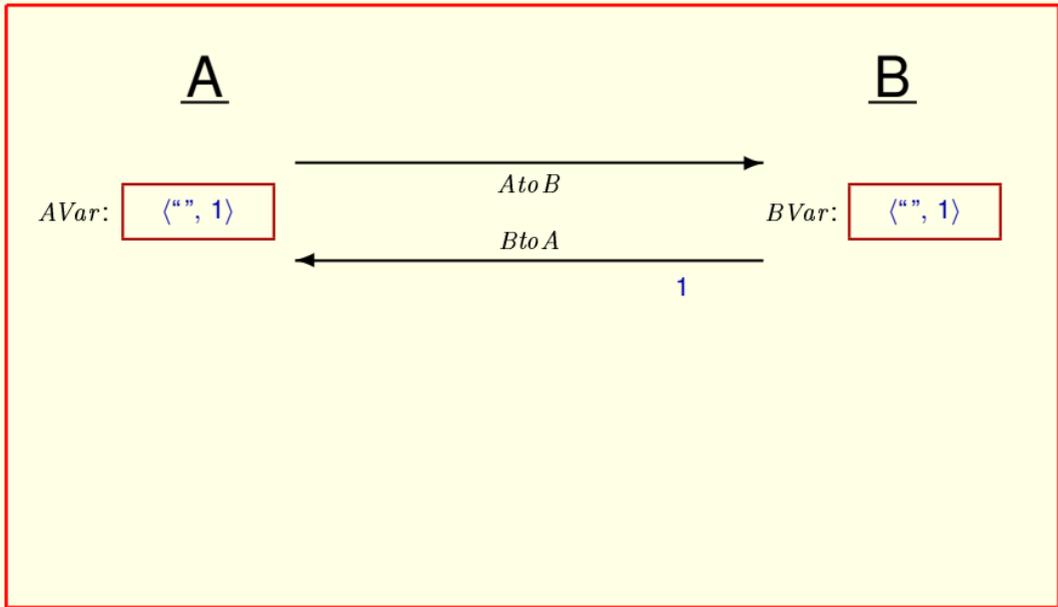
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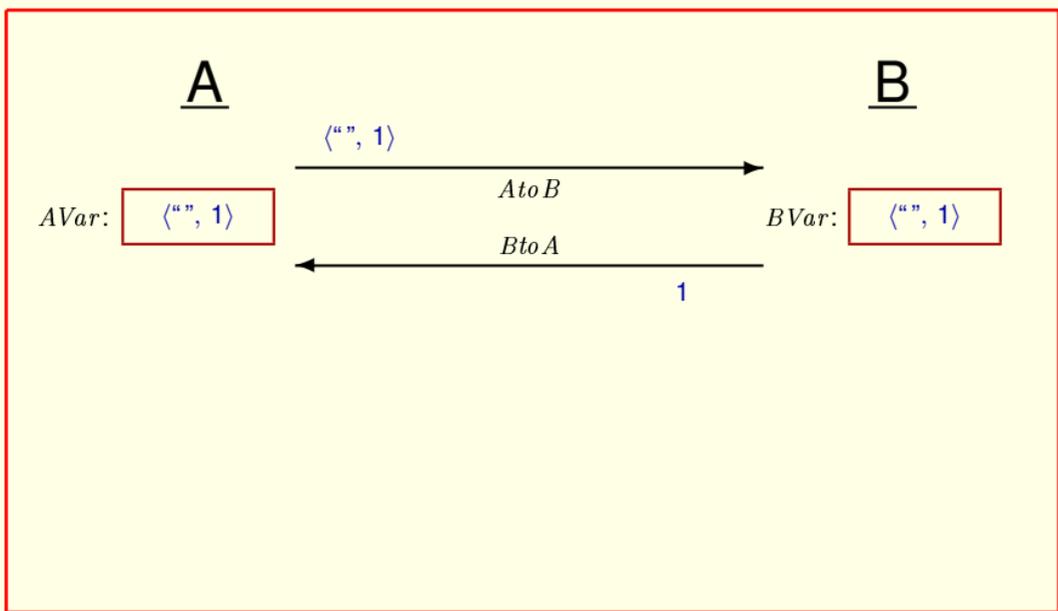
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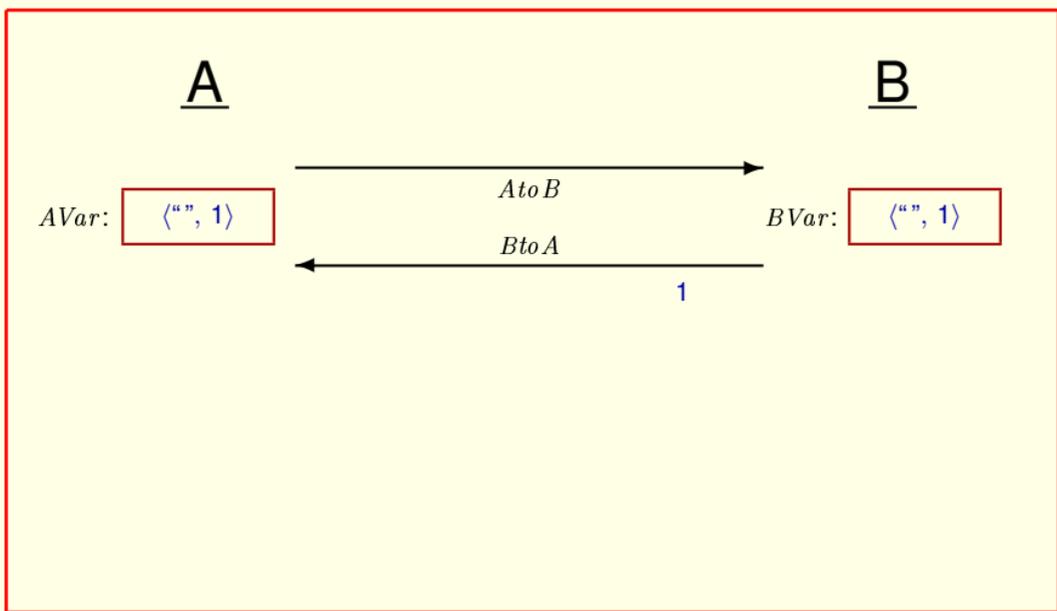
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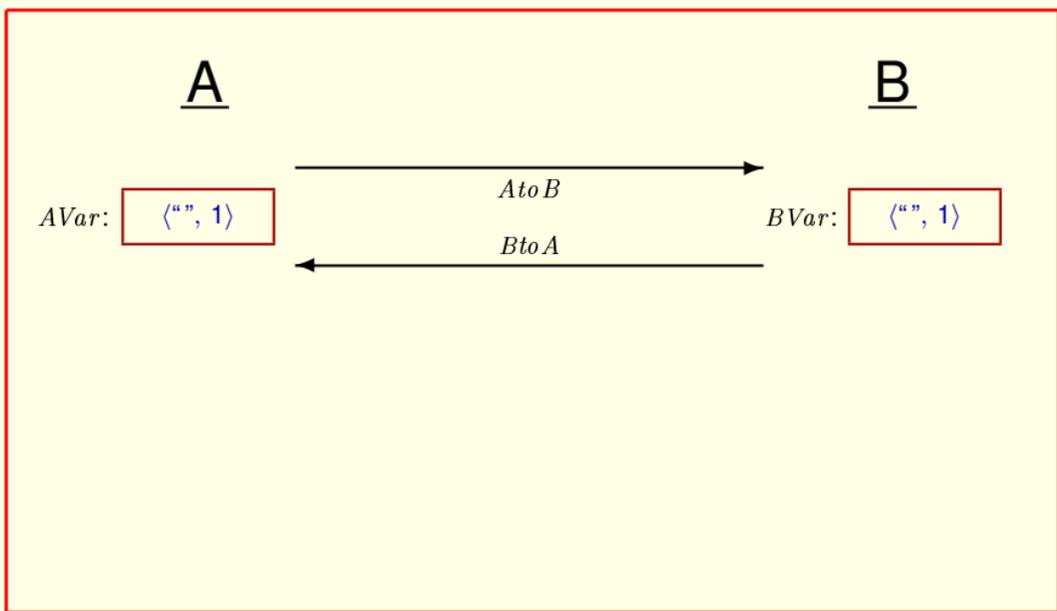
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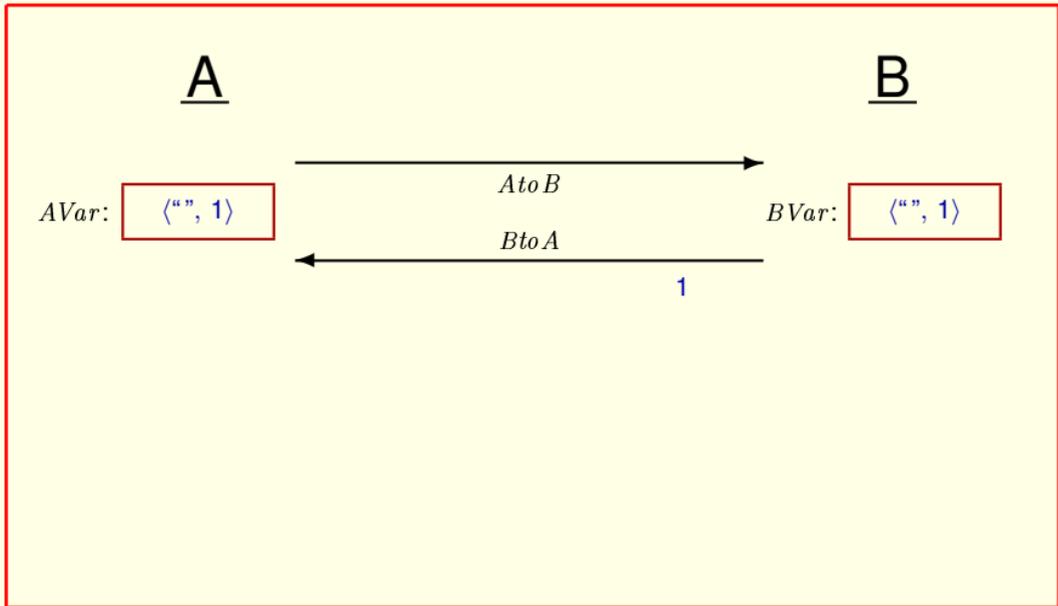
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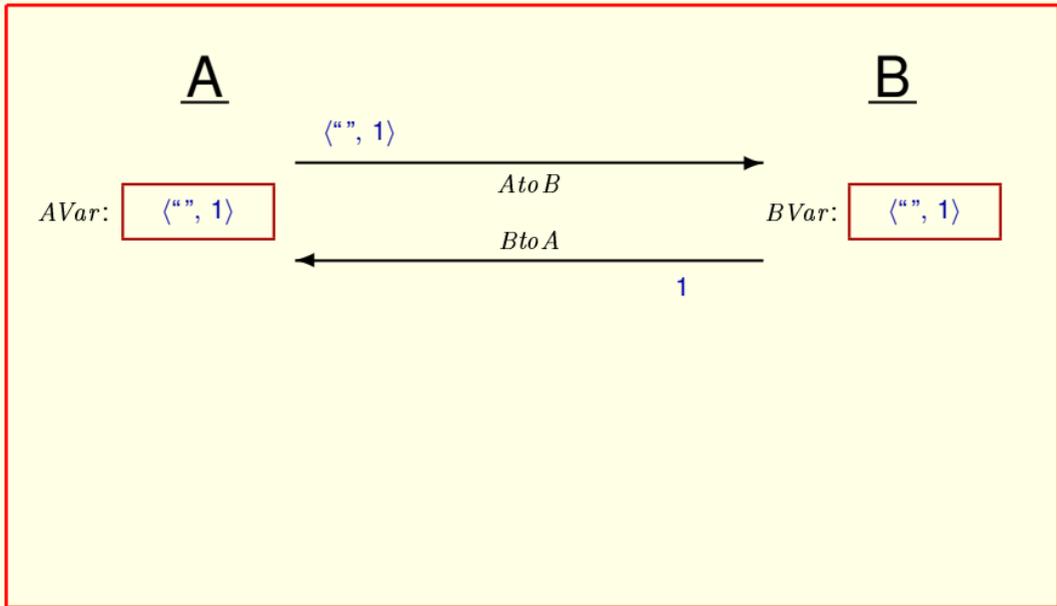
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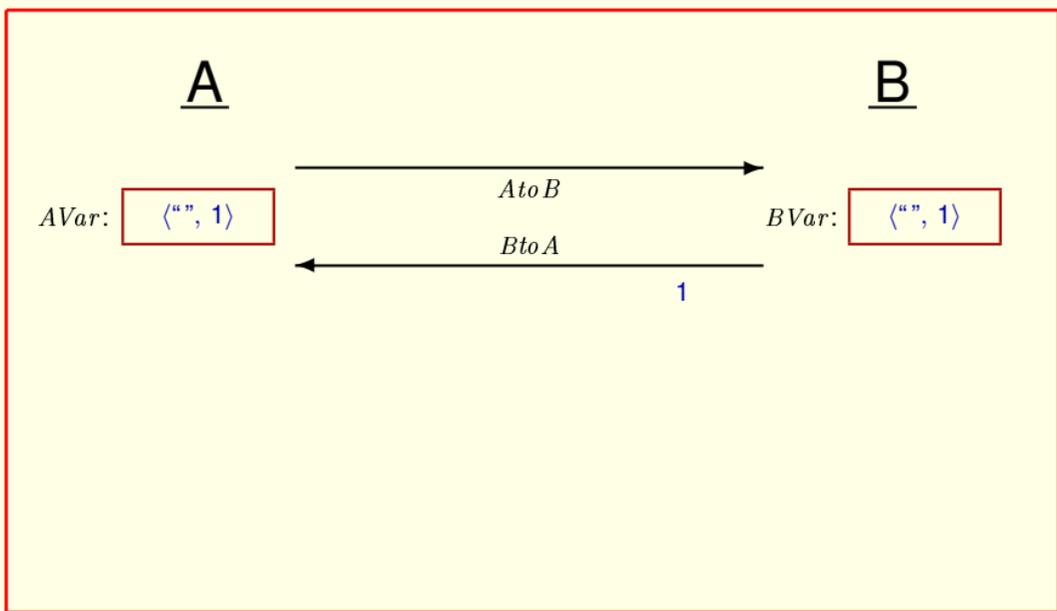
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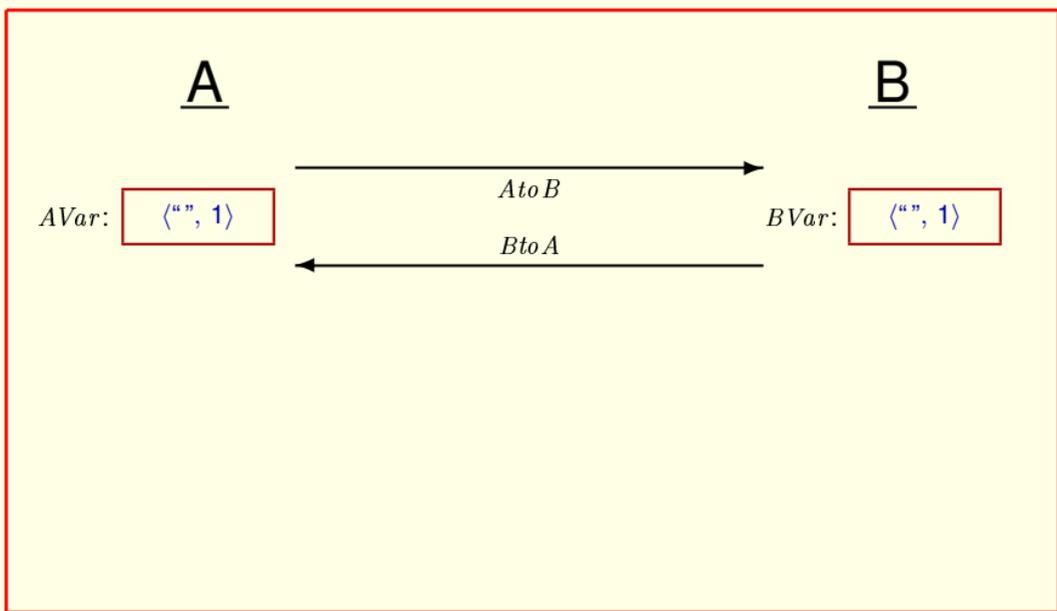
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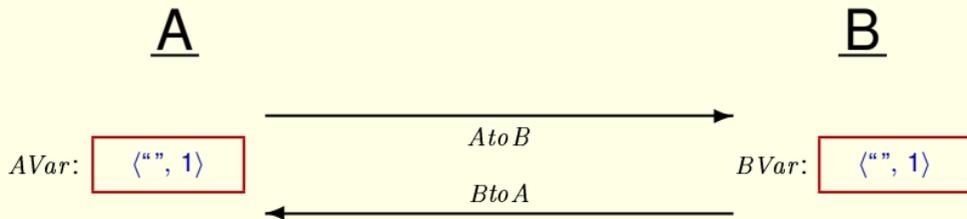
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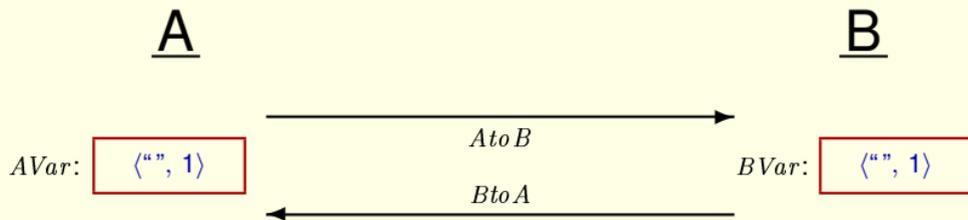
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And this continues forever.



$WF_{vars}(ASnd)$  and  $WF_{vars}(BSnd)$  are true  
because  $ASnd$  and  $BSnd$  steps keep occurring.

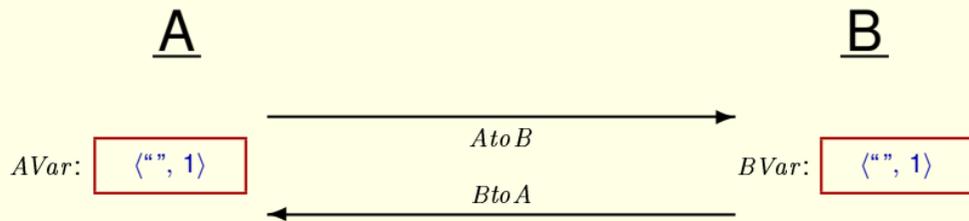
Weak fairness of A-send and B-send are true for this behavior because  
A-send and B-send steps keep occurring.



What about  $WF_{vars}(ARcv)$ ?

Weak fairness of A-send and B-send are true for this behavior because A-send and B-send steps keep occurring.

What about weak fairness of A-receive?



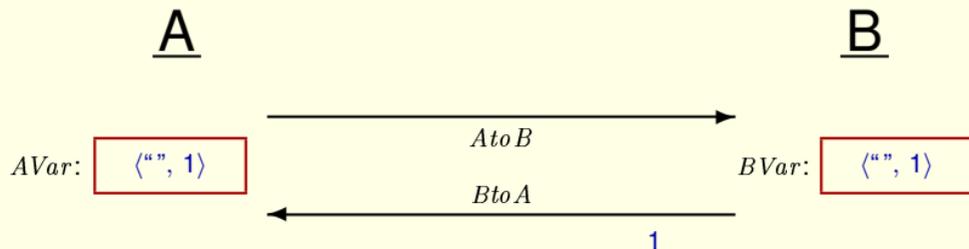
$ARcv$ : not enabled

What about  $WF_{vars}(ARcv)$ ?

Weak fairness of A-send and B-send are true for this behavior because A-send and B-send steps keep occurring.

What about weak fairness of A-recv?

A-recv is not enabled in the initial state, since  $BtoA$  contains no messages.



$ARcv:$     enabled

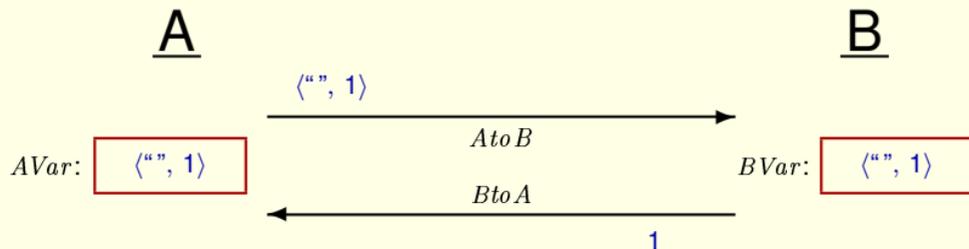
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Weak fairness of A-send and B-send are true for this behavior because A-send and B-send steps keep occurring.

What about weak fairness of A-recv?

A-recv is not enabled in the initial state, since  $BtoA$  contains no messages.

It becomes enabled when  $B$  sends a message.



$ARcv:$  enabled

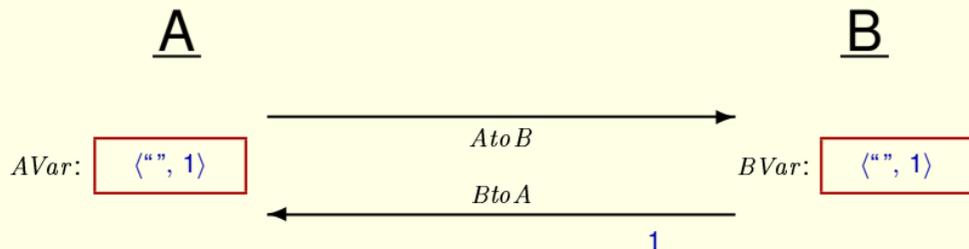
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$ARcv$ : enabled

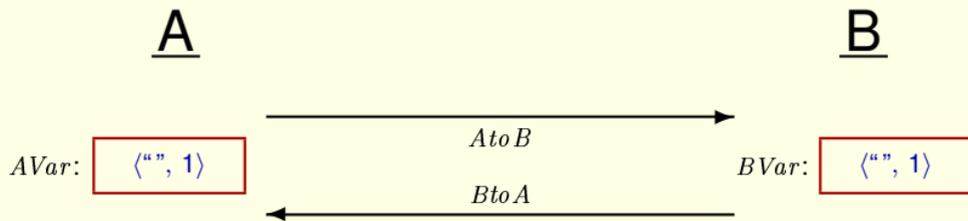
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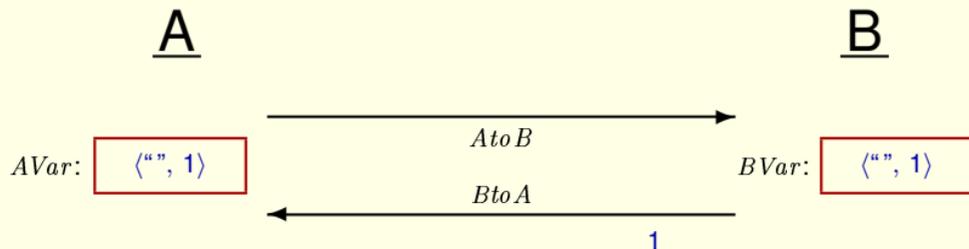
It becomes enabled when  $B$  sends a message.



$ARcv$ : not enabled

What about  $WF_{vars}(ARcv)$ ?

It becomes disabled when that message is lost.

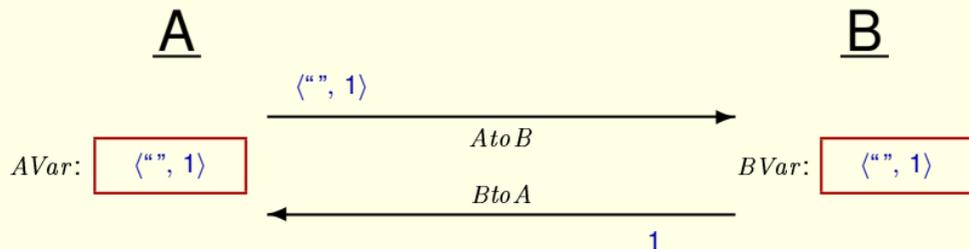


$ARcv:$  enabled

What about  $WF_{vars}(ARcv)$ ?

It becomes disabled when that message is lost.

It becomes enabled again when  $B$  sends another message.

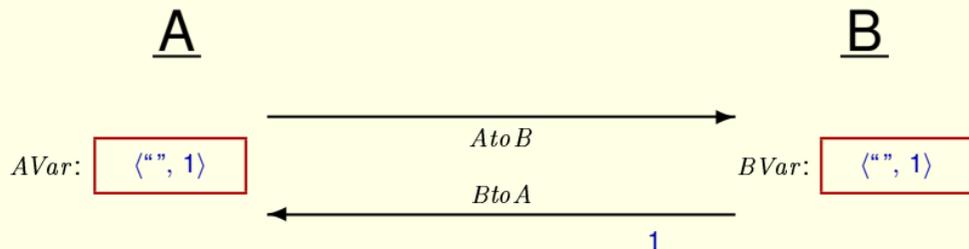


$ARcv:$     enabled

What about  $WF_{vars}(ARcv)$ ?

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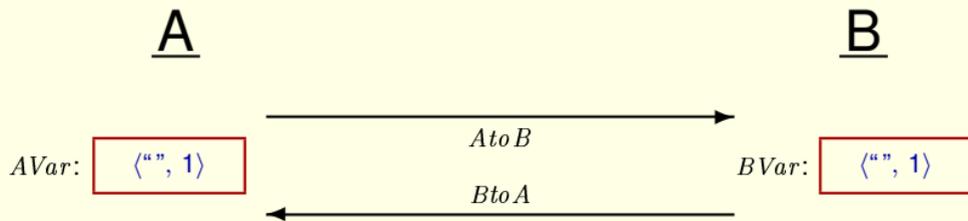


$ARcv:$     enabled

What about  $WF_{vars}(ARcv)$ ?

It becomes disabled when that message is lost.

It becomes enabled again when  $B$  sends another message.



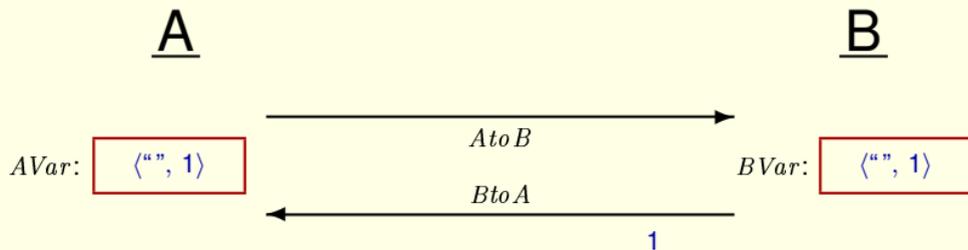
$ARcv$ : not enabled

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It becomes enabled again when  $B$  sends another message.

It is disabled again when that message is lost.



$ARcv$ : enabled

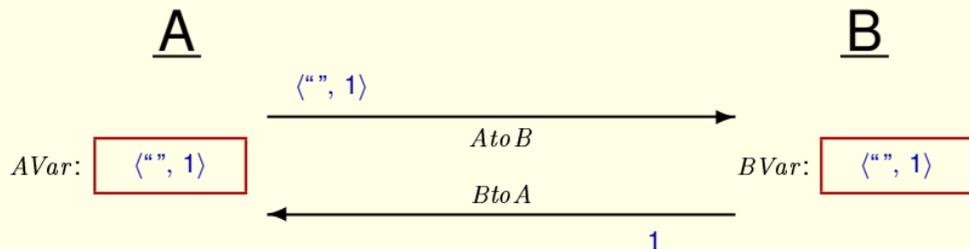
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It is disabled again when that message is lost.

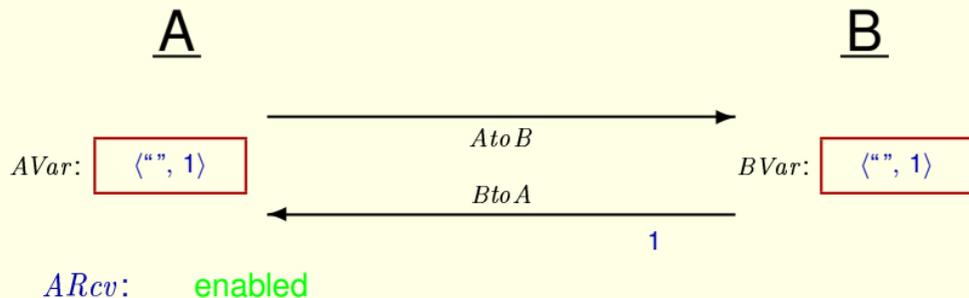
It becomes enabled again when  $B$  sends yet another message.



$ARcv:$     enabled

What about  $WF_{vars}(ARcv)$  ?

- It becomes disabled when that message is lost.
- It becomes enabled again when  $B$  sends another message.
- It is disabled again when that message is lost.
- It becomes enabled again when  $B$  sends yet another message.



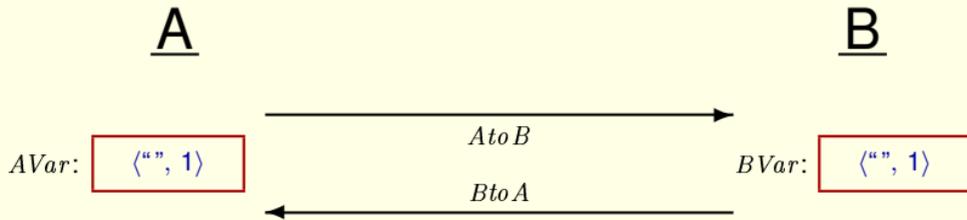
What about  $WF_{vars}(ARcv)$ ?

It becomes disabled when that message is lost.

It becomes enabled again when  $B$  sends another message.

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$ARcv$ : not enabled

What about  $WF_{vars}(ARcv)$ ?

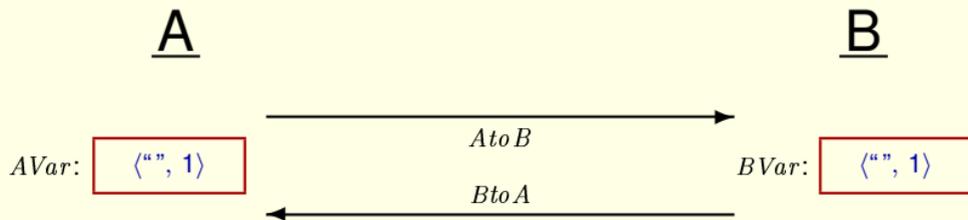
It becomes disabled when that message is lost.

It becomes enabled again when  $B$  sends another message.

It is disabled again when that message is lost.

It becomes enabled again when  $B$  sends yet another message.

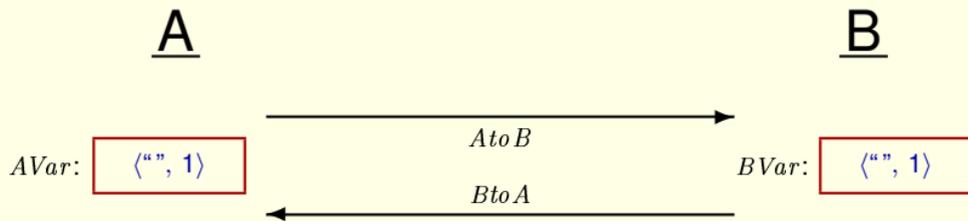
It's disabled again when that message is lost. And so on.



*ARcv*: not enabled

What about  $WF_{vars}(ARcv)$ ?

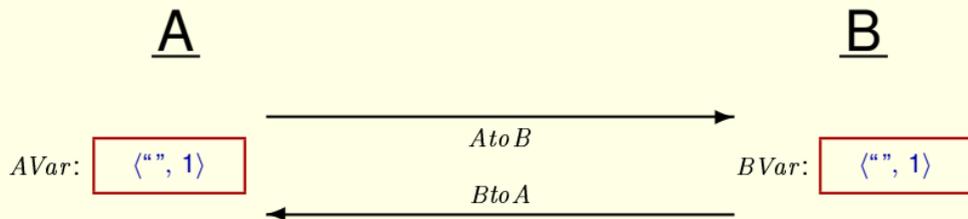
So weak fairness of A-receive



*ARcv*: not enabled

What about  $WF_{vars}(ARcv)$ ? True

So weak fairness of A-receive is true on this behavior

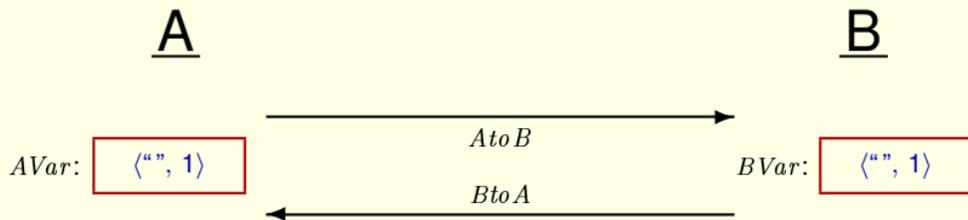


$ARcv$ : not enabled

What about  $WF_{vars}(ARcv)$ ? True  
because  $ARcv$  never continuously enabled.

So weak fairness of A-receive is true on this behavior

because A-receive keeps getting disabled after it's enabled, and it's never continuously enabled.



*ARcv*: not enabled

$WF_{vars}(BRcv)$  is also true.

So weak fairness of A-receive is true on this behavior

because A-receive keeps getting disabled after it's enabled, and it's never continuously enabled.

Weak fairness of B-receive is also true on this behavior for the same reason.

The behavior satisfies  $FairSpec$ , defined by:

$$FairSpec \triangleq Spec \wedge WF_{vars}(ARcv) \wedge WF_{vars}(BRcv) \wedge \\ WF_{vars}(ASnd) \wedge WF_{vars}(BSnd)$$

The behavior satisfies  $FairSpec$ , when it's defined like this.

The behavior satisfies  $FairSpec$ , defined by:

$$FairSpec \triangleq Spec \wedge WF_{vars}(ARcv) \wedge WF_{vars}(BRcv) \wedge \\ WF_{vars}(ASnd) \wedge WF_{vars}(BSnd)$$

but doesn't satisfy  $ABS!FairSpec$ .

The behavior satisfies  $FairSpec$ , when it's defined like this.

but it doesn't satisfy the high level fair spec in module  $ABSpec$  because no values are ever sent from A to B.

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~~THEOREM  $FairSpec \Rightarrow ABS!FairSpec$~~

The behavior satisfies  $FairSpec$ , when it's defined like this.

but it doesn't satisfy the high level fair spec in module  $ABSspec$  because no values are ever sent from A to B.

So this theorem is not true.

$$FairSpec \triangleq Spec \wedge WF_{vars}(ARcv) \wedge WF_{vars}(BRcv) \wedge \\ WF_{vars}(ASnd) \wedge WF_{vars}(BSnd)$$

The behavior satisfies *FairSpec* , when it's defined like this.

but it doesn't satisfy the high level fair spec in module *ABSpec* because no values are ever sent from A to B.

**So this theorem is not true.**

$$\text{FairSpec} \triangleq \text{Spec} \wedge \text{WF}_{\text{vars}}(\text{ARcv}) \wedge \text{WF}_{\text{vars}}(\text{BRcv}) \wedge \\ \text{WF}_{\text{vars}}(\text{ASnd}) \wedge \text{WF}_{\text{vars}}(\text{BSnd})$$

The problem is that

$$\text{FairSpec} \triangleq \text{Spec} \wedge \boxed{\text{WF}_{\text{vars}}(\text{ARcv})} \wedge \boxed{\text{WF}_{\text{vars}}(\text{BRcv})} \wedge \text{WF}_{\text{vars}}(\text{ASnd}) \wedge \text{WF}_{\text{vars}}(\text{BSnd})$$

Don't imply *ARcv* or *BRcv* steps ever occur,  
because actions keep getting disabled.

The problem is that

these weak fairness conditions don't imply that any A-receive or B-receive steps ever occur, because those actions keep getting disabled.

Weak fairness of action  $A$  asserts of a behavior:

If  $A$  ever remains continuously enabled,  
then an  $A$  step must eventually occur.

Remember that weak fairness of  $A$  means if  $A$  ever remains continuously enabled, then an  $A$  step must eventually occur.

Strong

~~Weak~~ fairness of action  $A$  asserts of a behavior:

is repeatedly

If  $A$  ever ~~remains continuously~~ enabled,  
then an  $A$  step must eventually occur.

Remember that weak fairness of  $A$  means if  $A$  ever remains continuously enabled, then an  $A$  step must eventually occur.

*Strong* fairness of  $A$  means that if  $A$  ever *is repeatedly* enabled, then an  $A$  step must eventually occur.

Strong

~~Weak~~ fairness of action  $A$  asserts of a behavior:

is repeatedly

If  $A$  ever ~~remains continuously~~ enabled,  
then an  $A$  step must eventually occur.

$\dots \rightarrow s_{42} \rightarrow s_{43} \rightarrow s_{44} \rightarrow s_{45} \rightarrow s_{46} \rightarrow s_{47} \rightarrow s_{48} \rightarrow s_{49} \rightarrow s_{50} \rightarrow \dots$

For example, suppose we have a behavior,

Strong

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If  $A$  ever ~~remains continuously~~ enabled,  
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$A$  enabled:

For example, suppose we have a behavior, and  $A$  enabled is

Strong

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$A$  enabled: false

For example, suppose we have a behavior, and  $A$  enabled is false in this state,

Strong

~~Weak~~ fairness of action  $A$  asserts of a behavior:

~~is repeatedly~~  
If  $A$  ever ~~remains continuously~~ enabled,  
then an  $A$  step must eventually occur.

$\dots \rightarrow s_{42} \rightarrow s_{43} \rightarrow s_{44} \rightarrow s_{45} \rightarrow s_{46} \rightarrow s_{47} \rightarrow s_{48} \rightarrow s_{49} \rightarrow s_{50} \rightarrow \dots$

$A$  enabled: false true

For example, suppose we have a behavior, and  $A$  enabled is false in this state, then true,

Strong

~~Weak~~ fairness of action  $A$  asserts of a behavior:

is repeatedly

If  $A$  ever ~~remains continuously~~ enabled,  
then an  $A$  step must eventually occur.

$\dots \rightarrow s_{42} \rightarrow s_{43} \rightarrow s_{44} \rightarrow s_{45} \rightarrow s_{46} \rightarrow s_{47} \rightarrow s_{48} \rightarrow s_{49} \rightarrow s_{50} \rightarrow \dots$

$A$  enabled: false true false

For example, suppose we have a behavior, and  $A$  enabled is false in this state, then true, the false again,

Strong

~~Weak~~ fairness of action  $A$  asserts of a behavior:

~~is repeatedly~~  
If  $A$  ever ~~remains continuously~~ enabled,  
then an  $A$  step must eventually occur.

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$A$  enabled: false true false true

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$A$  enabled: false true false true false true

For example, suppose we have a behavior, and  $A$  enabled is false in this state, then true, the false again, then true, then false and so on,

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$A$  enabled: false true false true false true false false true

where it keeps being re-enabled after it becomes disabled.

Then an  $A$  step must eventually occur.

Strong

~~Weak~~ fairness of action  $A$  asserts of a behavior:

~~is repeatedly~~  
If  $A$  ever ~~remains continuously~~ enabled,  
then an  $A$  step must eventually occur.

$\dots \rightarrow s_{42} \rightarrow s_{43} \rightarrow s_{44} \rightarrow s_{45} \rightarrow s_{46} \rightarrow s_{47} \rightarrow s_{48} \rightarrow s_{49} \rightarrow s_{50} \rightarrow \dots$

$A$  enabled: false true false true false true false false true

Or equivalently:

$A$  cannot be repeatedly enabled forever  
without another  $A$  step occurring.

where it keeps being re-enabled after it becomes disabled.

Then an  $A$  step must eventually occur.

An equivalent way of saying this is that  $A$  cannot be repeatedly enabled forever without another  $A$  step occurring.

$$\text{FairSpec} \triangleq \text{Spec} \wedge \text{WF}_{\text{vars}}(\text{ARcv}) \wedge \text{WF}_{\text{vars}}(\text{BRcv}) \wedge \\ \text{WF}_{\text{vars}}(\text{ASnd}) \wedge \text{WF}_{\text{vars}}(\text{BSnd})$$

We need to change the definition of *FairSpec* to what it was originally

$$FairSpec \triangleq Spec \wedge \boxed{WF_{vars}(ARcv)} \wedge \boxed{WF_{vars}(BRcv)} \wedge \\ WF_{vars}(ASnd) \wedge WF_{vars}(BSnd)$$

We need to change the definition of *FairSpec* to what it was originally  
changing these weak fairness conditions

$$FairSpec \triangleq Spec \wedge \boxed{SF_{vars}(ARcv)} \wedge \boxed{SF_{vars}(BRcv)} \wedge \\ \boxed{WF_{vars}(ASnd)} \wedge \boxed{WF_{vars}(BSnd)}$$

We need to change the definition of *FairSpec* to what it was originally changing these weak fairness conditions **to strong fairness**.

$$FairSpec \triangleq Spec \wedge \mathbf{SF}_{vars}(ARcv) \wedge \mathbf{SF}_{vars}(BRcv) \wedge \\ \mathbf{WF}_{vars}(ASnd) \wedge \mathbf{WF}_{vars}(BSnd)$$

*B* must keep sending messages

We need to change the definition of *FairSpec* to what it was originally changing these weak fairness conditions to strong fairness.

Since the *B*-send action is always enabled, weak fairness of *B*-send implies that *B* keeps sending messages.

$$FairSpec \triangleq Spec \wedge \boxed{SF_{vars}(ARcv)} \wedge SF_{vars}(BRcv) \wedge \\ WF_{vars}(ASnd) \wedge \boxed{WF_{vars}(BSnd)}$$

*B* must keep sending messages  
which implies *A* must eventually  
receive those messages.

We need to change the definition of *FairSpec* to what it was originally changing these weak fairness conditions to strong fairness.

Since the *B*-send action is always enabled, weak fairness of *B*-send implies that *B* keeps sending messages. This keeps enabling *A*-receive which, by strong fairness implies that *A*-receive steps must eventually occur to receive those messages — even if *Loss*-message actions keep disabling *A*-receive.

$$FairSpec \stackrel{\Delta}{=} Spec \wedge SF_{vars}(ARcv) \wedge SF_{vars}(BRcv) \wedge$$
$$\boxed{WF_{vars}(ASnd)} \wedge WF_{vars}(BSnd)$$

*A* must keep sending messages

Similarly, *A* must keep sending messages

$$\text{FairSpec} \stackrel{\Delta}{=} \text{Spec} \wedge \text{SF}_{\text{vars}}(\text{ARcv}) \wedge \text{SF}_{\text{vars}}(\text{BRcv}) \wedge \text{WF}_{\text{vars}}(\text{ASnd}) \wedge \text{WF}_{\text{vars}}(\text{BSnd})$$

*A* must keep sending messages  
that *B* must eventually receive.

Similarly, *A* must keep sending messages that *B* must eventually receive.

$$\text{FairSpec} \triangleq \text{Spec} \wedge \mathbf{SF}_{\text{vars}}(\text{ARcv}) \wedge \mathbf{SF}_{\text{vars}}(\text{BRcv}) \wedge \\ \mathbf{WF}_{\text{vars}}(\text{ASnd}) \wedge \mathbf{WF}_{\text{vars}}(\text{BSnd})$$

Similarly, *A* must keep sending messages that *B* must eventually receive.

**With this definition,**

$$\text{FairSpec} \stackrel{\Delta}{=} \text{Spec} \wedge \mathbf{SF}_{\text{vars}}(\text{ARcv}) \wedge \mathbf{SF}_{\text{vars}}(\text{BRcv}) \wedge \\ \mathbf{WF}_{\text{vars}}(\text{ASnd}) \wedge \mathbf{WF}_{\text{vars}}(\text{BSnd})$$

**THEOREM**  $\text{FairSpec} \Rightarrow \text{ABS!FairSpec}$

Similarly,  $A$  must keep sending messages that  $B$  must eventually receive.

With this definition, the theorem is true.

$$\text{FairSpec} \stackrel{\Delta}{=} \text{Spec} \wedge \text{SF}_{\text{vars}}(\text{ARcv}) \wedge \text{SF}_{\text{vars}}(\text{BRcv}) \wedge \\ \text{WF}_{\text{vars}}(\text{ASnd}) \wedge \text{WF}_{\text{vars}}(\text{BSnd})$$

THEOREM  $\text{FairSpec} \Rightarrow \text{ABS!FairSpec}$

TLC will now find no error.

Similarly,  $A$  must keep sending messages that  $B$  must eventually receive.

With this definition, the theorem is true.

You can change the definition of  $\text{FairSpec}$  in the module and rerun the model, and TLC will now find no error.

# What Good is Liveness?

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What good is knowing that something eventually happens?

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## What Good is Liveness?

What good is knowing that something eventually happens – if it could be  $10^6$  years from now?

What Good is Liveness?

What good is knowing that something eventually happens?  
If it could be a million years from now when it happens.

## What Good is Liveness?

What good is knowing that something eventually happens – if it could be  $10^6$  years from now?

How can we ensure strong fairness of the *ARcv* and *BRcv* actions?

How can we ensure strong fairness of the *ARcv* and *BRcv* actions?

## What Good is Liveness?

What good is knowing that something eventually happens – if it could be  $10^6$  years from now?

How can we ensure strong fairness of the *ARcv* and *BRcv* actions? Or ever know that it's not satisfied?

How can we ensure strong fairness of the *ARcv* and *BRcv* actions?  
Or ever know that it's not satisfied? Since it would take forever to be sure that it's not.

A specification is an abstraction.

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It's a compromise between our desires for accuracy and simplicity.

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We'd like to require that a message is received within 4.7 ms.

A specification is an abstraction.

It's a compromise between our desires for accuracy and simplicity.

We'd like to require that a message is received within 4.7 milliseconds of when it's sent.

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We'd like to require that a message is received within 4.7 ms.

**But that would require specifying:**

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We'd like to require that a message is received within 4.7 ms.

But that would require specifying:

- How long it can take a message to be received.

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How long it can take a message to be received.

A specification is an abstraction.

It's a compromise between our desires for accuracy and simplicity.

We'd like to require that a message is received within 4.7 ms.

**But that would require specifying:**

- How long it can take a message to be received.
- How often messages can be lost.

But that would require specifying:

How long it can take a message to be received.

How often messages can be lost.

A specification is an abstraction.

It's a compromise between our desires for accuracy and simplicity.

We'd like to require that a message is received within 4.7 ms.

**But that would require specifying:**

- How long it can take a message to be received.
- How often messages can be lost.
- **How frequently messages are retransmitted.**

But that would require specifying:

How long it can take a message to be received.

How often messages can be lost.

**And how frequently messages are retransmitted.**

It's simpler to require that a message is eventually received.

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It's simpler to require that a message is eventually received.

If it's not eventually received, it can't be received within 4.7 ms.

It's simpler to require that a message is eventually received.

And if it's not eventually received, it certainly can't be received within 4.7 milliseconds.

It's simpler to require that a message is eventually received.

If it's not eventually received, it can't be received within 4.7 ms.

For systems without hard real-time response requirements,

It's simpler to require that a message is eventually received.

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For systems without hard real-time response requirements,

It's simpler to require that a message is eventually received.

If it's not eventually received, it can't be received within 4.7 ms.

For systems without hard real-time response requirements, liveness checking is a useful way to find errors that prevent things from happening.

It's simpler to require that a message is eventually received.

And if it's not eventually received, it certainly can't be received within 4.7 milliseconds.

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In the first eight lectures, you learned about writing the safety part of a TLA+ spec. Now you know how to specify liveness. You simply add weak and strong fairness conditions. Simple, yes. Easy, no. Liveness is inherently subtle. TLA+ is the simplest way I know to express it, and it's still hard.

But don't worry if you have trouble with liveness. The safety part is by far the largest part and almost always the most important part of a spec. A major reason to add liveness is to catch errors in the safety part. If your fairness conditions don't imply the eventually or leads-to properties you expect to hold, it could be because the safety part doesn't allow behaviors that it should.

[ slide 281 ]

**End of Lecture 9, Part 2**

**THE ALTERNATING BIT PROTOCOL**  
**THE PROTOCOL**